Meson-baryon interaction in the hadron exchange picture

M. Döring

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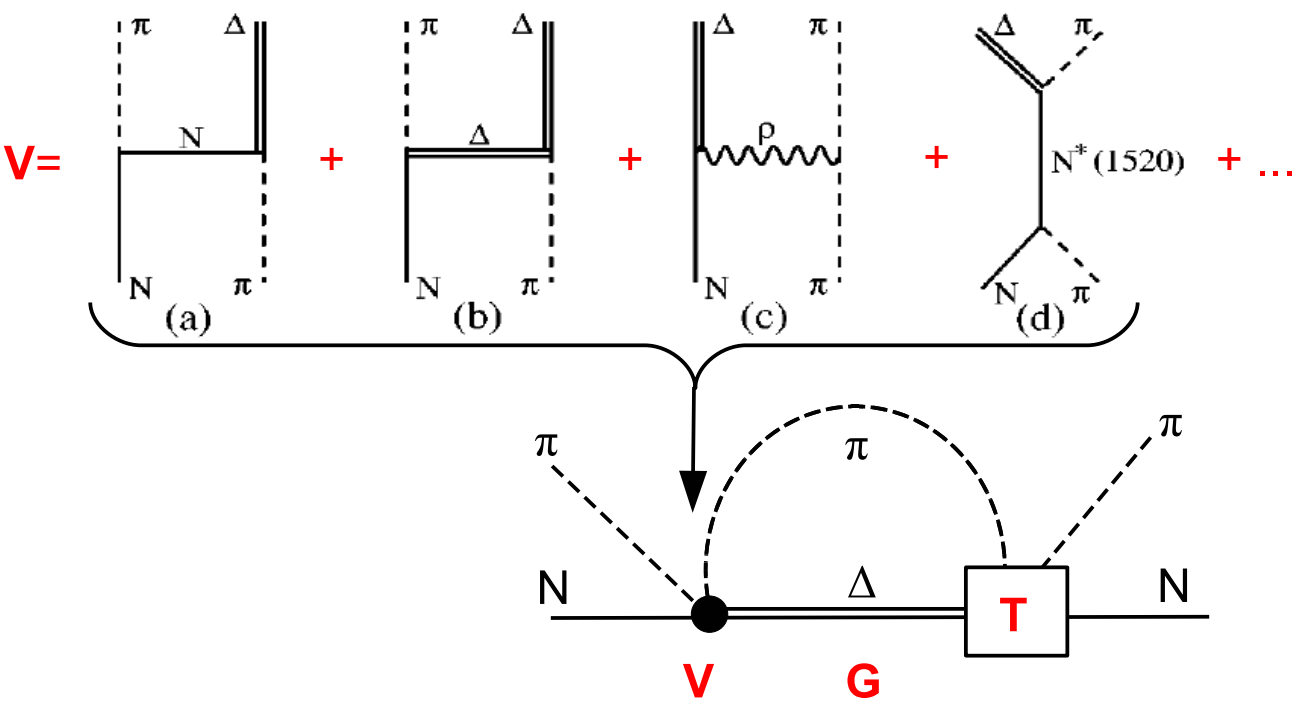
The Jülich model of pion-nucleon interaction

Motivation

- Coupled channels πN, ηN, KY; effective $\pi\pi N$ channels σN, ρN, $\pi\Delta$.
- Chiral Lagrangian of Wess and Zumino [PR163 (1967), Phys.Rept. 161 (1988)].
- Baryonic resonances up to $J = 7/2$ with derivative couplings.

Scattering equation in the JLS basis

$$\langle L'S'k'|T^{IJ}_{\mu\nu}|LSk\rangle = \langle L'S'k'|V^{IJ}_{\mu\nu}|LSk\rangle$$
$$+ \sum_{\gamma,L''S''} \int_0^\infty k''^2\, dk'' \langle L'S'k'|V^{IJ}_{\mu\gamma}|L''S''k''\rangle \frac{1}{Z - E_\gamma(k'') + i\epsilon} \langle L''S''k''|T^{IJ}_{\gamma\nu}|LSk\rangle$$



Scattering equation in the JLS basis

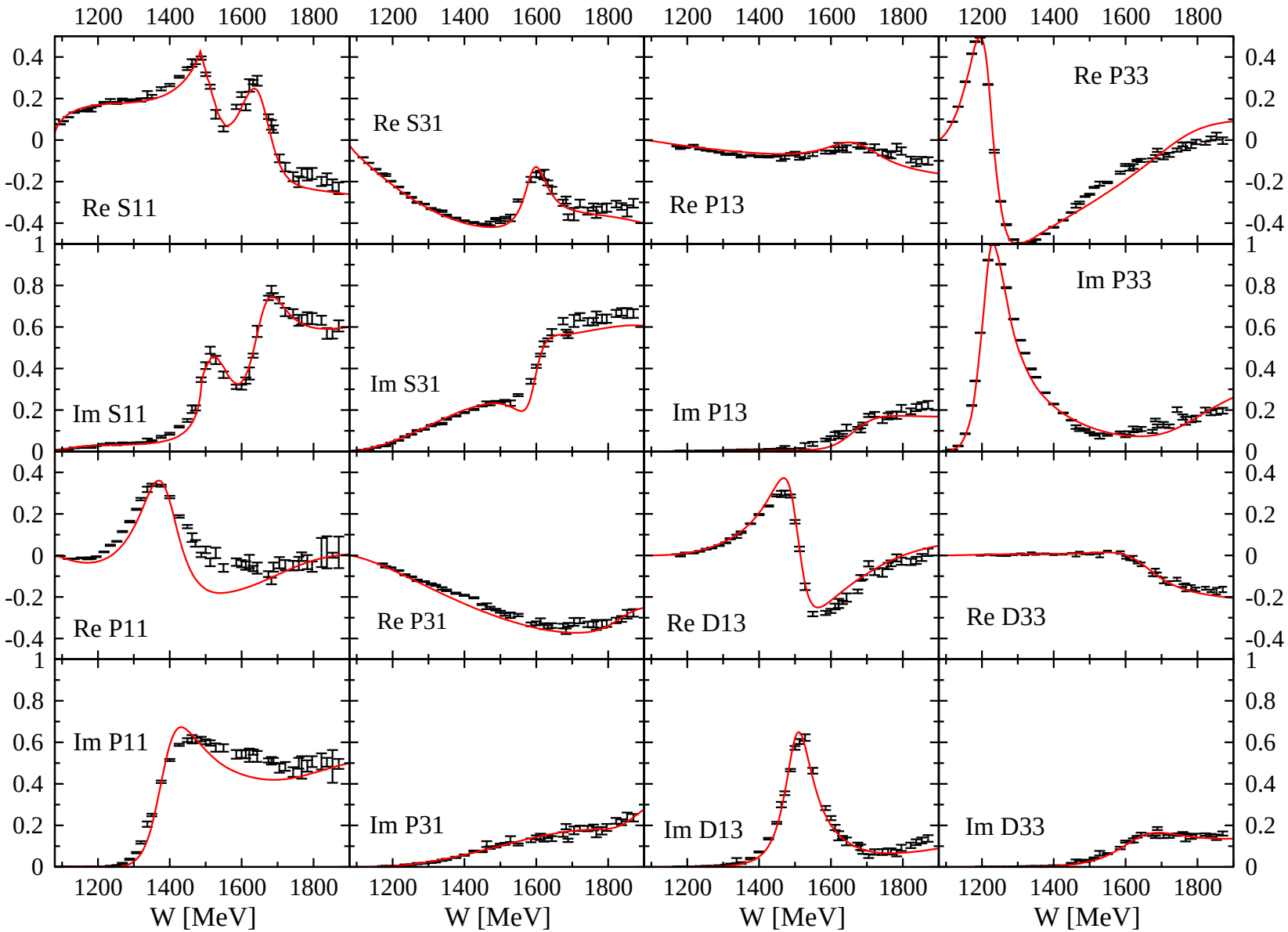
$$\langle L'S'k'|T^{IJ}_{\mu\nu}|LSk\rangle = \langle L'S'k'|V^{IJ}_{\mu\nu}|LSk\rangle$$
$$+ \sum_{\gamma,L''S''} \int_0^\infty k''^2\, dk'' \langle L'S'k'|V^{IJ}_{\mu\gamma}|L''S''k''\rangle \frac{1}{Z - E_\gamma(k'') + i\epsilon} \langle L''S''k''|T^{IJ}_{\gamma\nu}|LSk\rangle$$

Features

- Hadron exchange: provides the relevant dynamics.
- Full analyticity (dispersive parts).
- All partial waves are linked (t-, u-channel processes)
- Channels linked (SU(3) symmetry).
- Dynamical generation of resonances is possible, but not easy (strong constraints).

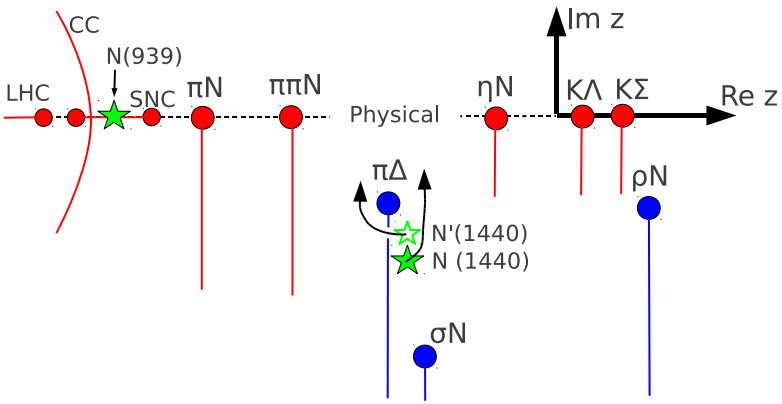
Partial waves in $\pi N \to \pi N$ (Solution 2002)

"Data": GWU/SAID, PRC74 (2006)



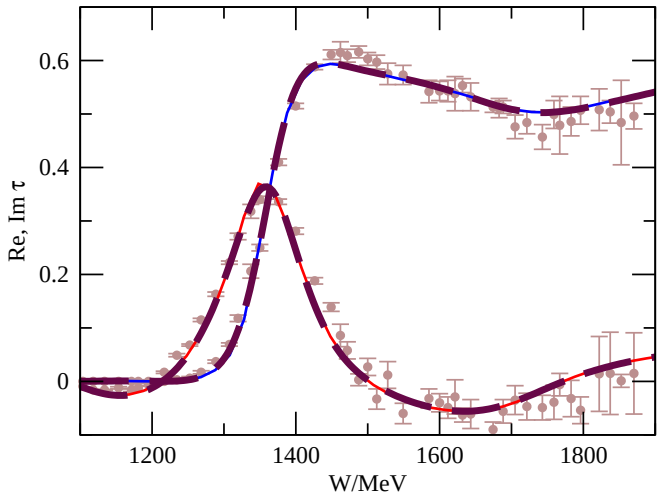
Structure of the P11 partial wave (Roper)

analytic continuation



- $N(\pi\pi)_{L=I=0}$ channel important (chiral σ); Roper dynamically generated in Jülich model.
- Roper pole $N(1440) + \pi\Delta$ branch point $\rightarrow$ non-standard resonance shape.
- Sub-threshold cuts from crossing symmetry.

- Where is the $3*$ N(1710)? [S. Ceci, M.D. et al, arXiv 1104.3490]



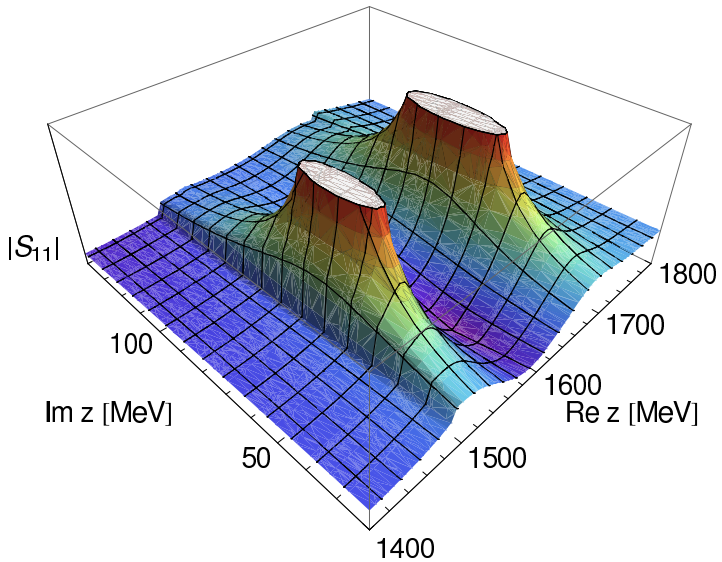
Fit of a model without ρN branch point (CMB type) [solid lines] to the Jülich amplitude [dashed lines]

- CMB fit to JM has pole at $1698 - 130\,i$ MeV, simulates missing branch point.
- $\rightarrow$ Inclusion of full analytic structure important to avoid false pole signals.

Branch points in $\gamma n \rightarrow \eta n$

Poles and residues

[M.D., C. Hanhart, F. Huang, S. Krewald and U.-G. Meißner, NPA 829 (2009), PLB 681 (2009)]



	Re z_0 [MeV]	-2 Im z_0 [MeV]	$\|R\|$ [MeV]	θ [deg] [^{0}]
$N^*(1440)\ P_{11}$	1387	147	48	-64
ARN	1359	162	38	-98
HOE	1385	164	40	
CUT	1375±30	180±40	52±5	-100±35
$N^*(1520)\ D_{13}$	1505	95	32	-18
ARN	1515	113	38	-5
HOE	1510	120	32	-8
CUT	1510±5	114±10	35±2	-12±5

	Re z_0 [MeV]	-2 Im z_0 [MeV]	$\|R\|$ [MeV]	θ [deg] [^{0}]
$N^*(1535)\ S_{11}$	1519	129	31	-3
ARN	1502	95	16	-16
HOE	1487			
CUT	1510±50	260±80	120±40	+15±45
$N^*(1650)\ S_{11}$	1669	136	54	-44
ARN	1648	80	14	-69
HOE	1670	163	39	-37
CUT	1640±20	150±30	60±10	-75±25
$N^*(1720)\ P_{13}$	1663	212	14	-82
ARN	1666	355	25	-94
HOE	1686	187	15	
CUT	1680±30	120±40	8±12	-160±30
$\Delta(1232)\ P_{33}$	1218	90	47	-37
ARN	1211	99	52	-47
HOE	1209	100	50	-48
CUT	1210±1	100±2	53±2	-47±1
$\Delta^*(1620)\ S_{31}$	1593	72	12	-108
ARN	1595	135	15	-92
HOE	1608	116	19	-95
CUT	1600±15	120±20	15±2	-110±20
$\Delta^*(1700)\ D_{33}$	1637	236	16	-38
ARN	1632	253	18	-40
HOE	1651	159	10	
CUT	1675±25	220±40	13±3	-20±25
$\Delta^*(1910)\ P_{31}$	1840	221	12	-153
ARN	1771	479	45	+172
HOE	1874	283	38	
CUT	1880±30	200±40	20±4	-90±30

[ARN]: Arndt et al., PRC 74 (2006), [HOE]: Höhler, πN Newsl. 9 (1993), [CUT]: Cutkowski et al., PRD 20 (1979).

Residues to ηN, σN, ρN, $\pi\Delta$. Zeros. Branching ratios to πN, ηN.

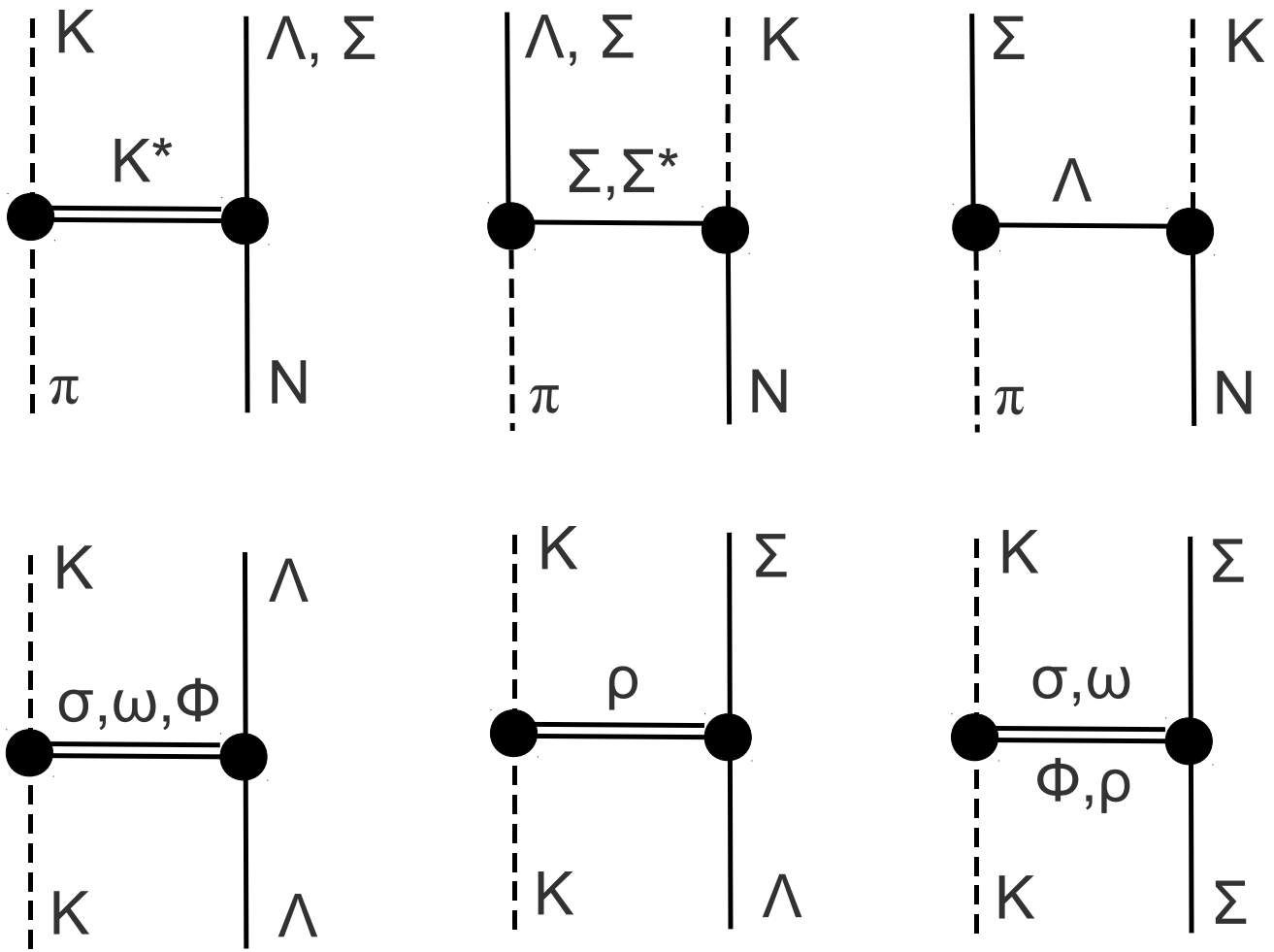
The reaction $\pi^+ p \to K^+ \Sigma^+$

Towards a unified analysis of different final states

- From the example of the $N(1710)P_{11}$: different reaction channels provide more information on resonance content than higher precision in one channel.
- SU(3) symmetry provides predictive power.
- $\pi^+ p \to K^+ \Sigma^+$: Pure isospin $3/2$; relatively few Δ resonances.
- Simultaneous fit to $\pi N \to \pi N$ partial waves [energy dependent GWU SAID solution, PRC74 (2006)] plus $\pi^+ p \to K^+ \Sigma^+$ observables.
- Guiding principle in the fit: Resonances are the last resort (while technically being the easiest way to improve the χ^2).
- Error analysis on pole positions is crucial.

Inclusion of the KY channels

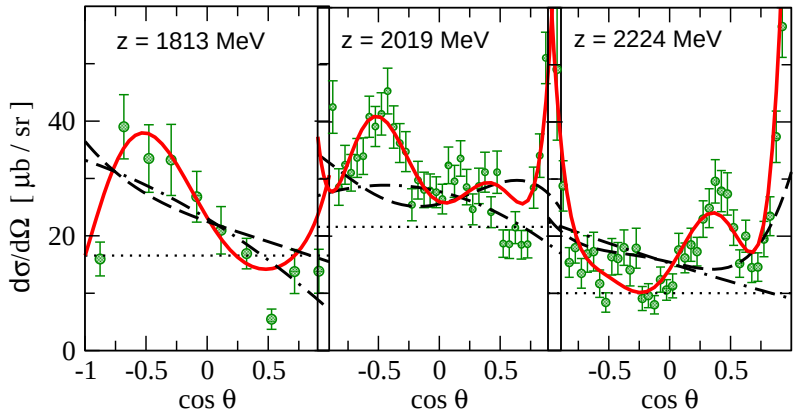
Inclusion via $SU(3)$ symmetry plus s-channel resonance vertices to KY (not shown; 40 parms.).

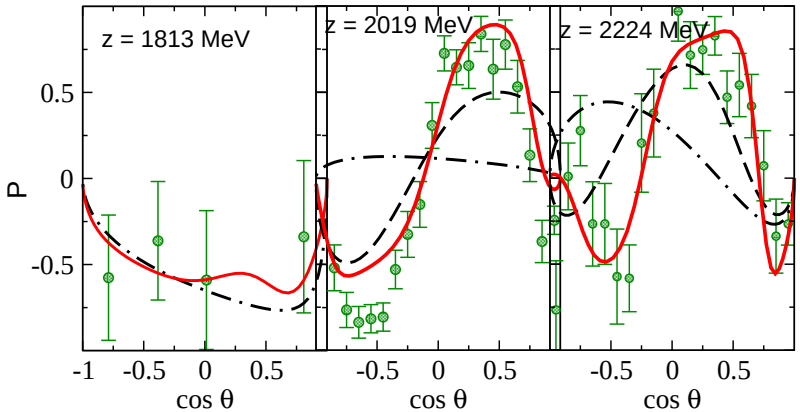


The reaction $\pi^+ p \to K^+ \Sigma^+$

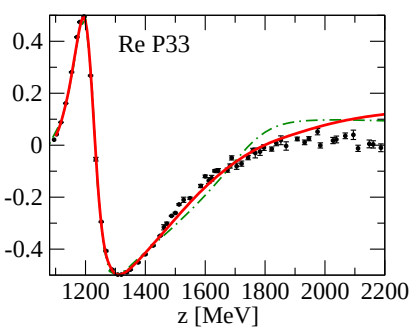
Link to full Results & Error analysis

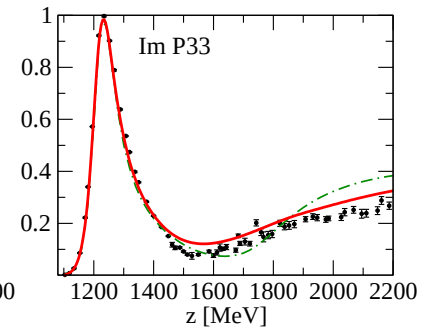
M.D., C. Hanhart, F. Huang, S. Krewald, U.-G. Meißner, D. Rönchen, [NPA851 (2011)]

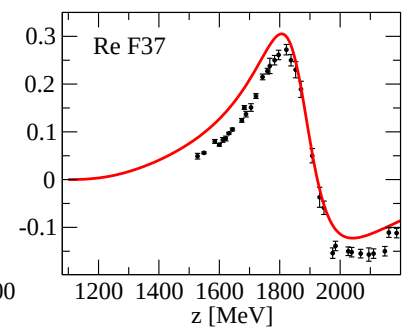


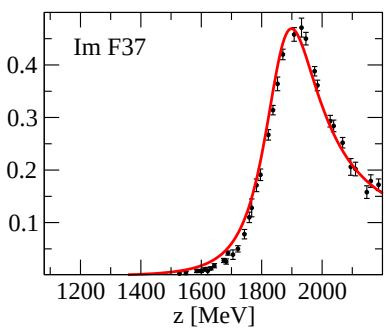


Data upper: Candlin 1983, NPB 226 (1983), lower: GWU/SAID, PRC74 (2006)



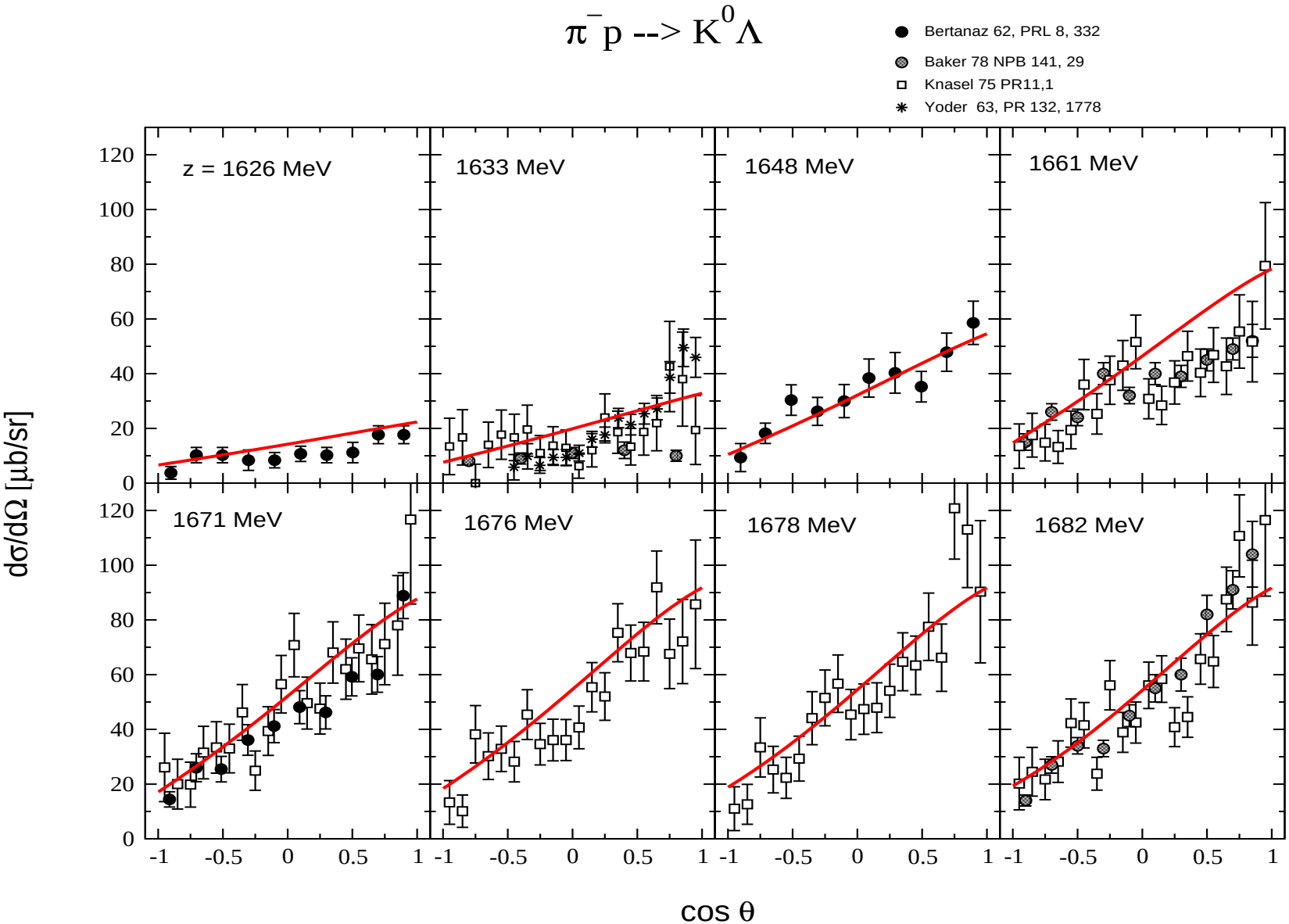


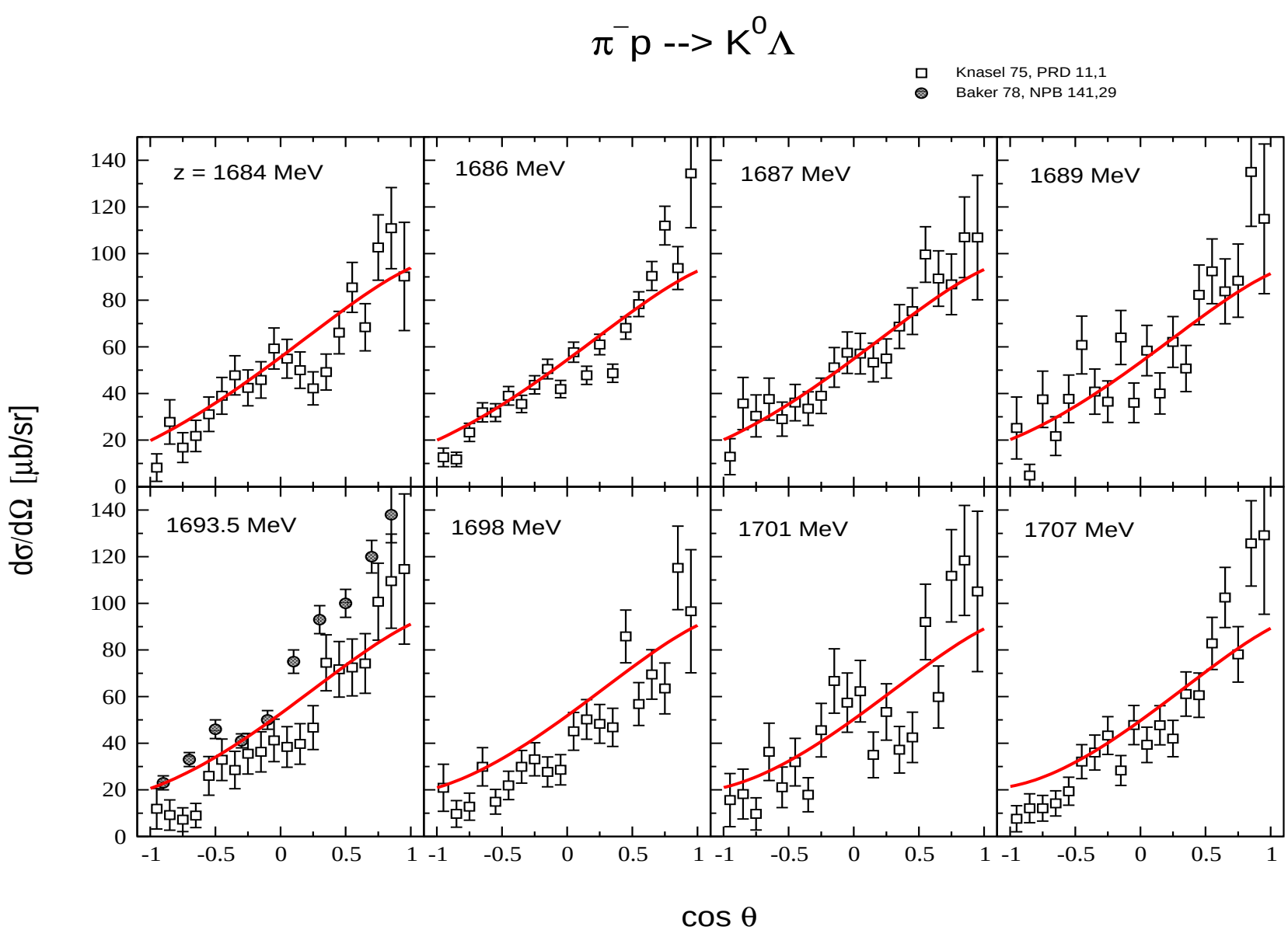


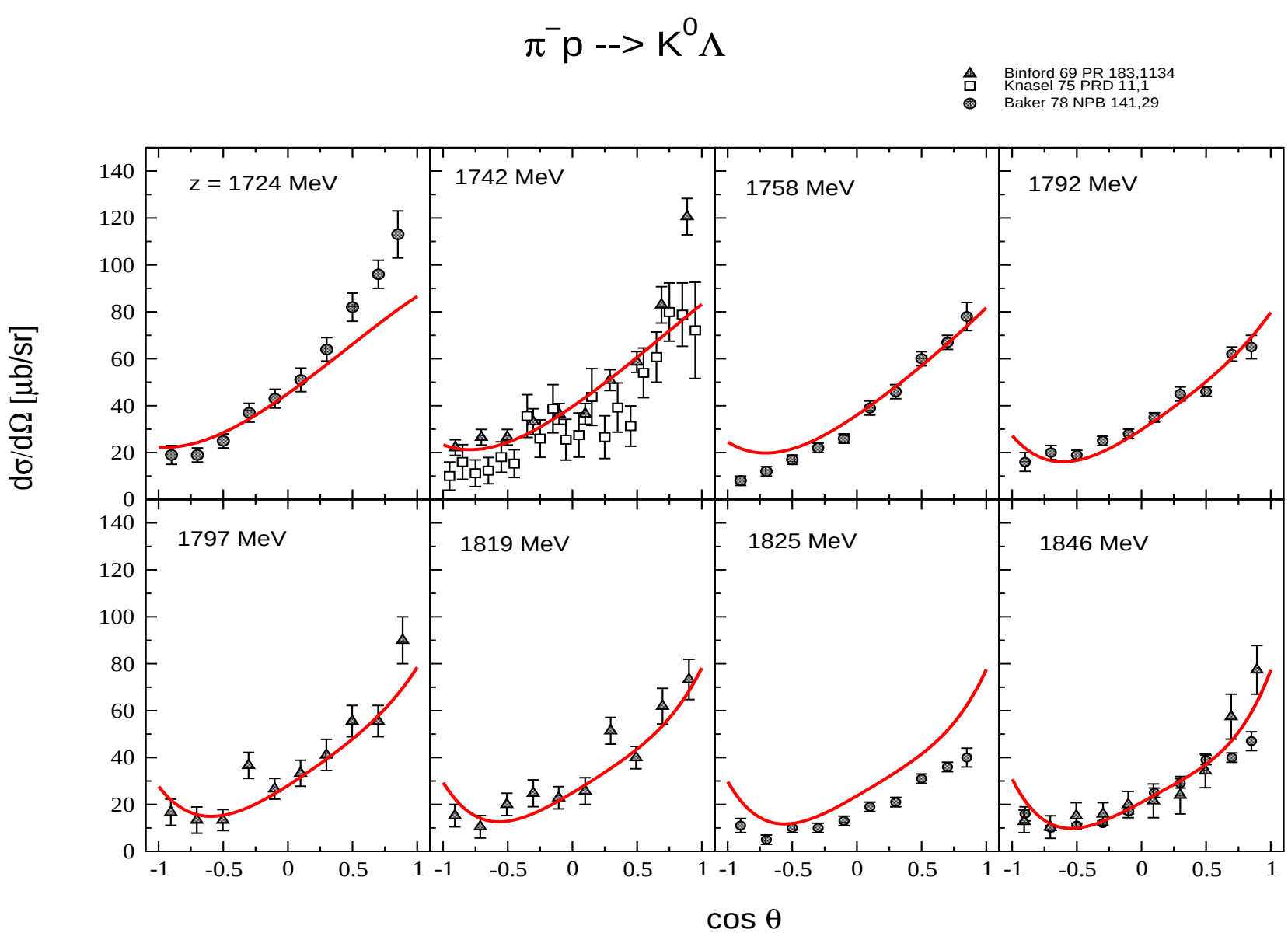


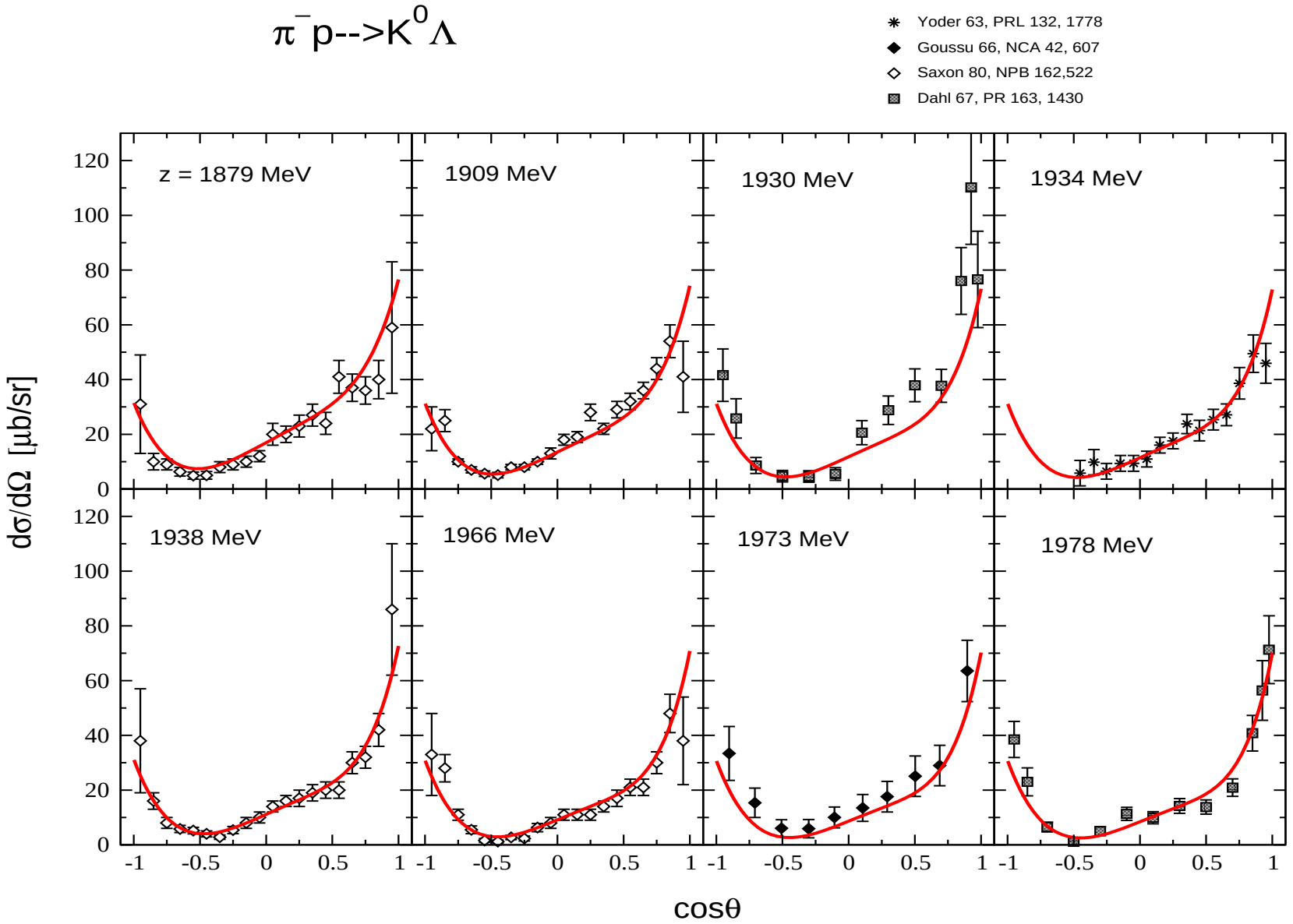
Linking partial waves and different reactions puts more constraints on resonance content.

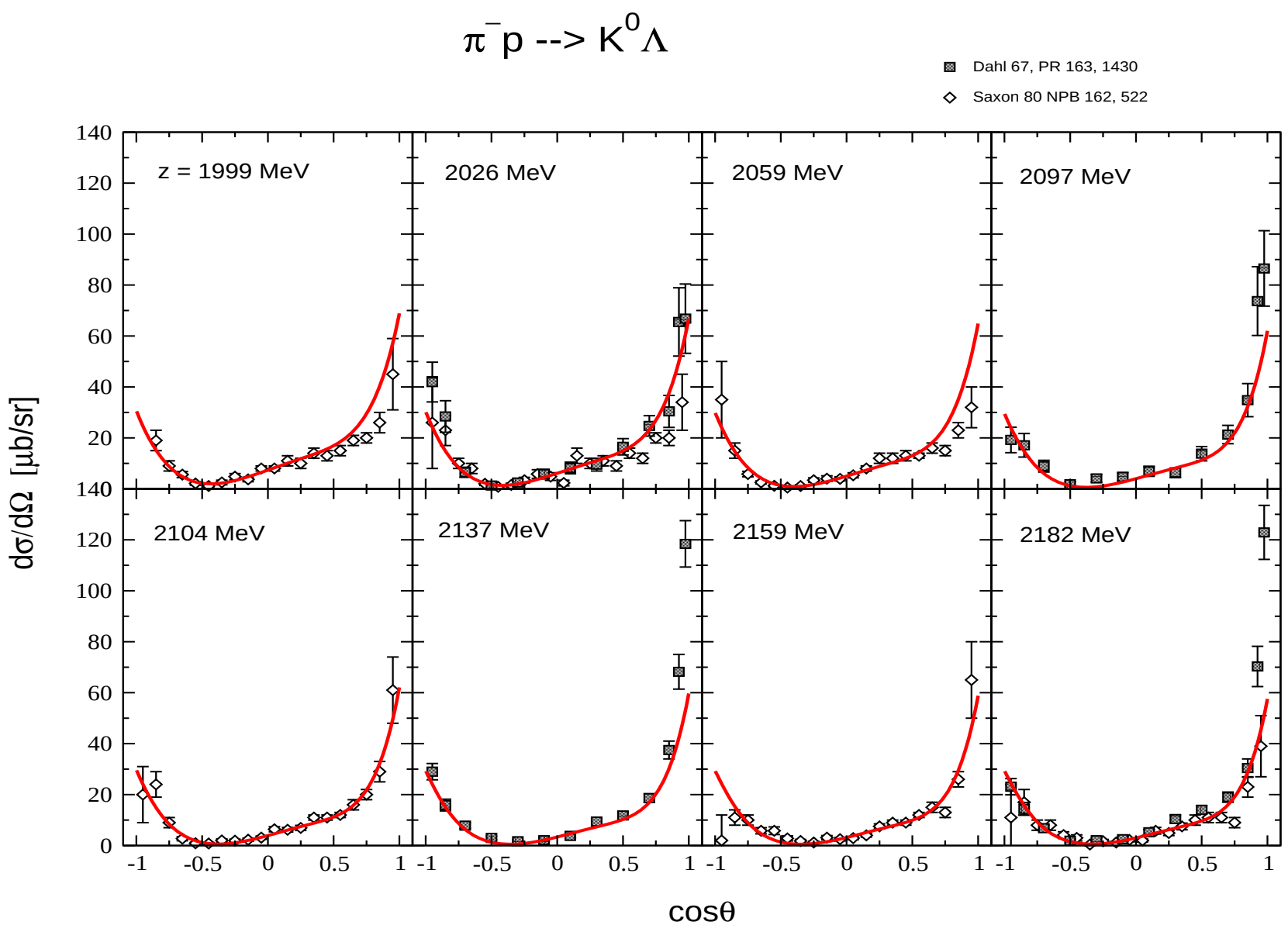
Example of other final states [preliminary, no N(1710)P11 needed so far]

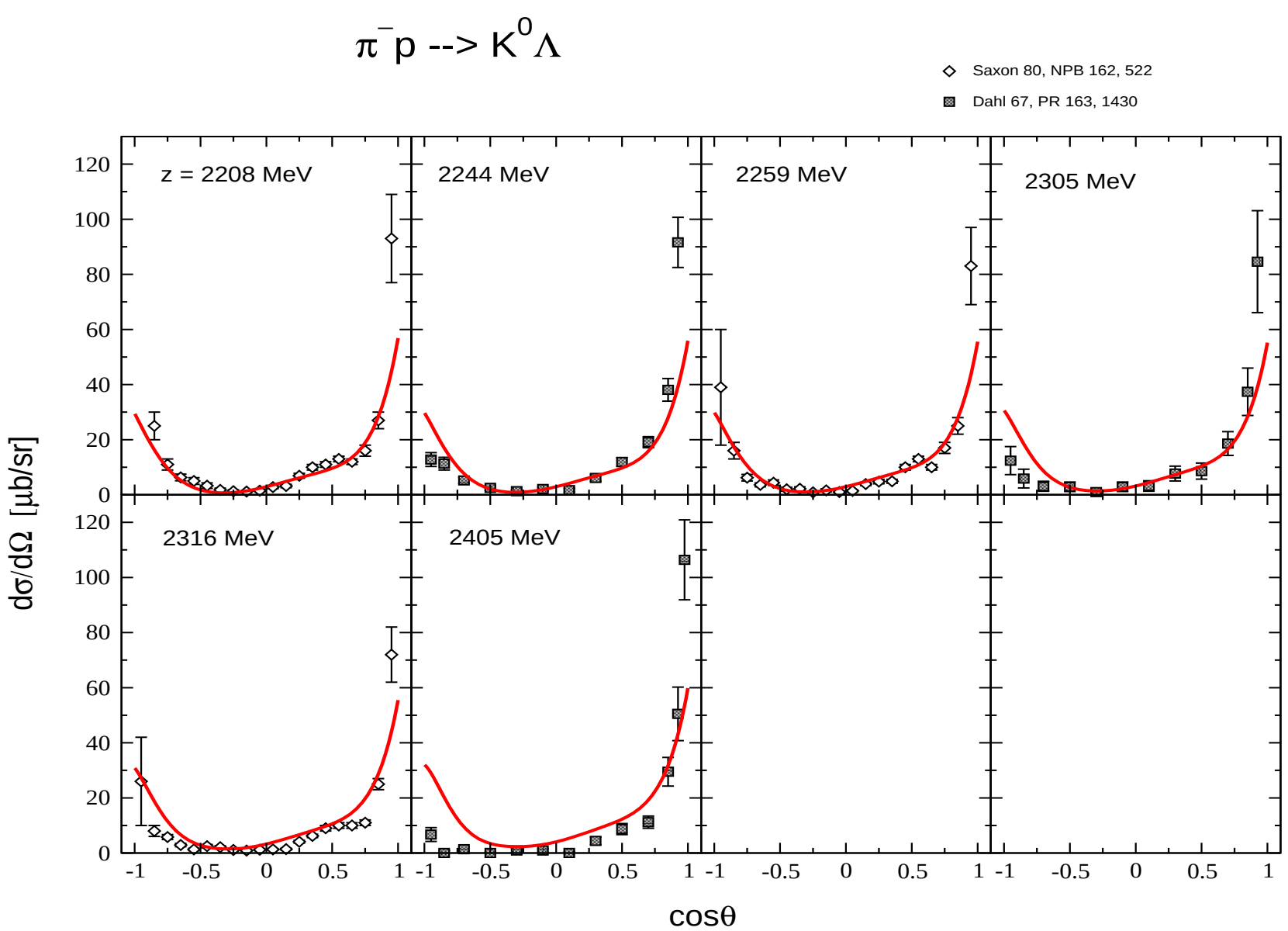



π⁻p --> K⁰Λ
Knasel 75, PRD 11,1
Baker 78, NPB 141,29
z = 1684 MeV
1686 MeV
1687 MeV
1689 MeV
1693.5 MeV
1698 MeV
1701 MeV
1707 MeV
dσ/dΩ [μb/sr]
cos θ


$\pi^- p \to K^0 \Lambda$
Binford 69 PR 183,1134
Knasel 75 PRD 11,1
Baker 78 NPB 141,29
z = 1724 MeV
1742 MeV
1758 MeV
1792 MeV
1797 MeV
1819 MeV
1825 MeV
1846 MeV
dσ/dΩ [μb/sr]
cos θ


$\pi^- p \to K^0 \Lambda$
Yoder 63, PRL 132, 1778
Goussu 66, NCA 42, 607
Saxon 80, NPB 162,522
Dahl 67, PR 163, 1430
z = 1879 MeV
1909 MeV
1930 MeV
1934 MeV
1938 MeV
1966 MeV
1973 MeV
1978 MeV
dσ/dΩ [μb/sr]
cosθ


$\pi^- p \dashrightarrow K^0 \Lambda$
Dahl 67, PR 163, 1430
Saxon 80 NPB 162, 522
z = 1999 MeV
2026 MeV
2059 MeV
2097 MeV
2104 MeV
2137 MeV
2159 MeV
2182 MeV
dσ/dΩ [μb/sr]
cosθ

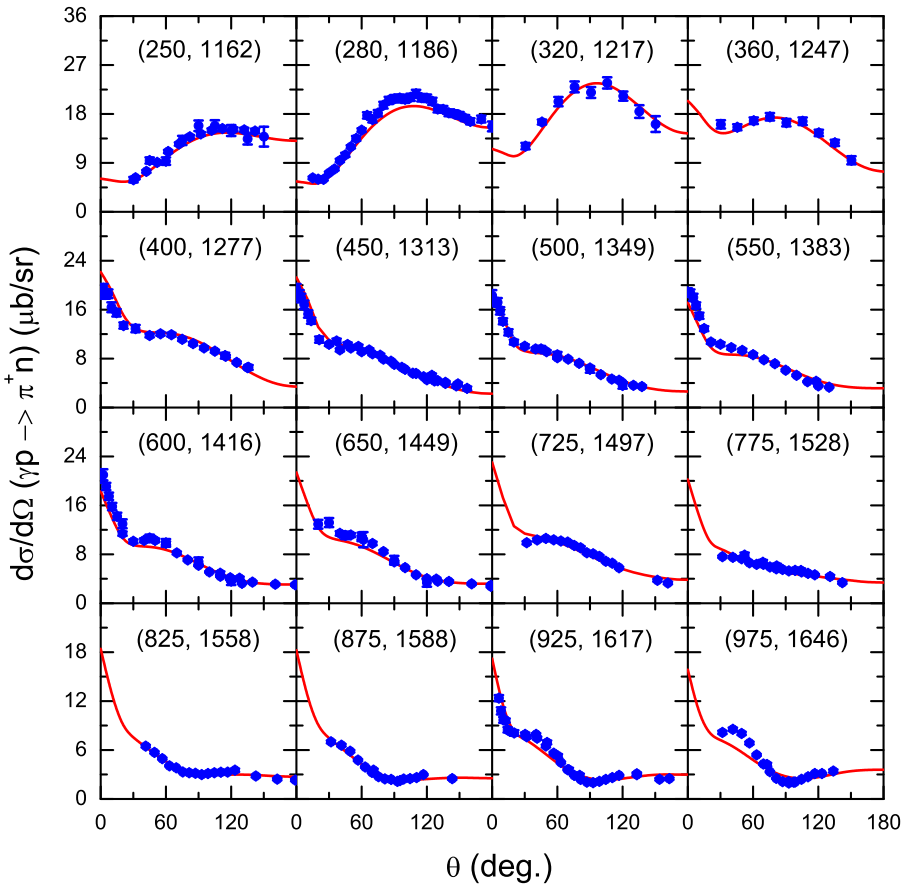

$\pi^- p \to K^0 \Lambda$
Saxon 80, NPB 162, 522
Dahl 67, PR 163, 1430
z = 2208 MeV
2244 MeV
2259 MeV
2305 MeV
2316 MeV
2405 MeV
dσ/dΩ [μb/sr]
cosθ

Linking reactions: $d\sigma/d\Omega$ and Σ_γ for $\gamma p \to \pi^+ n$

F. Huang, K. Nakayama, M.D., *et al.*, in prep. (see also arXiv:1103.2065)

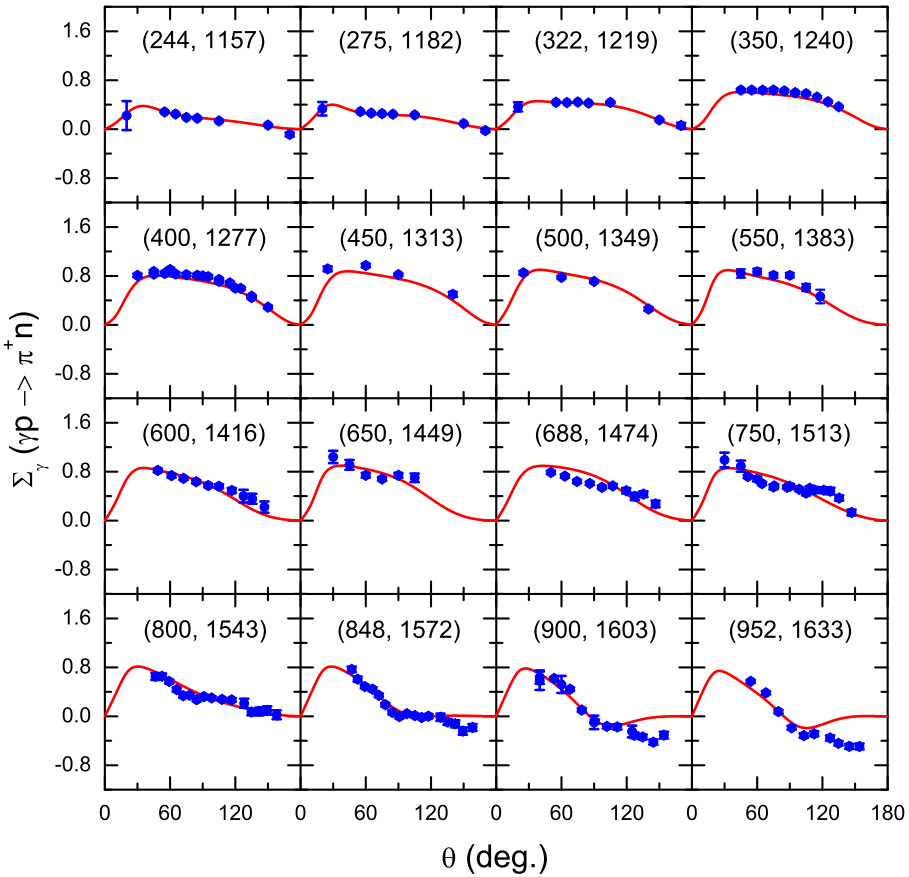
Gauge invariance respected

Further results $\gamma N \to \pi N$



Differential cross section for $\gamma p \to \pi^+ n$

Data: CNS Data analysis center [CBELSA/TAPS, JLAB, MAMI,...]



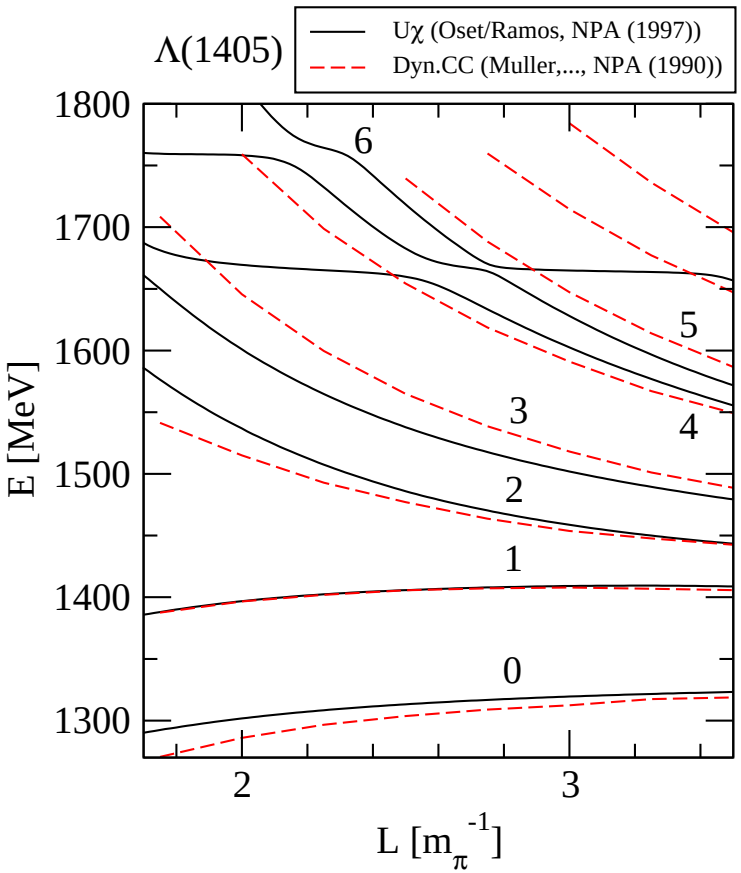
Photon spin asymmetry for $\gamma p \to \pi^+ n$

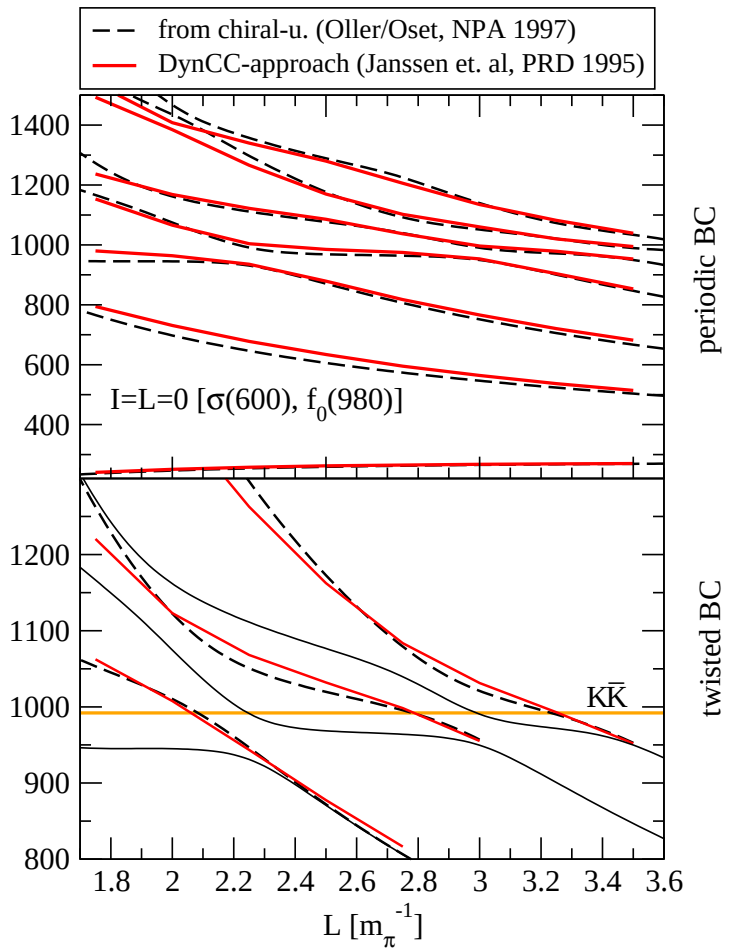
Dynamical coupled channels models in a box

Discretization & twisted BC

[M.D., J. Haidenbauer, A. Rusetsky, U.-G. Meißner, E. Oset, in preparation]

- Variation of box size $L \rightarrow$ reconstruction of phase shifts (Lüscher)
- Examples: $\Lambda(1405)$, $\sigma(600)$, $f_0(980)$ on the lattice.
- Prediction of lattice levels & including coming lattice-data in analysis.

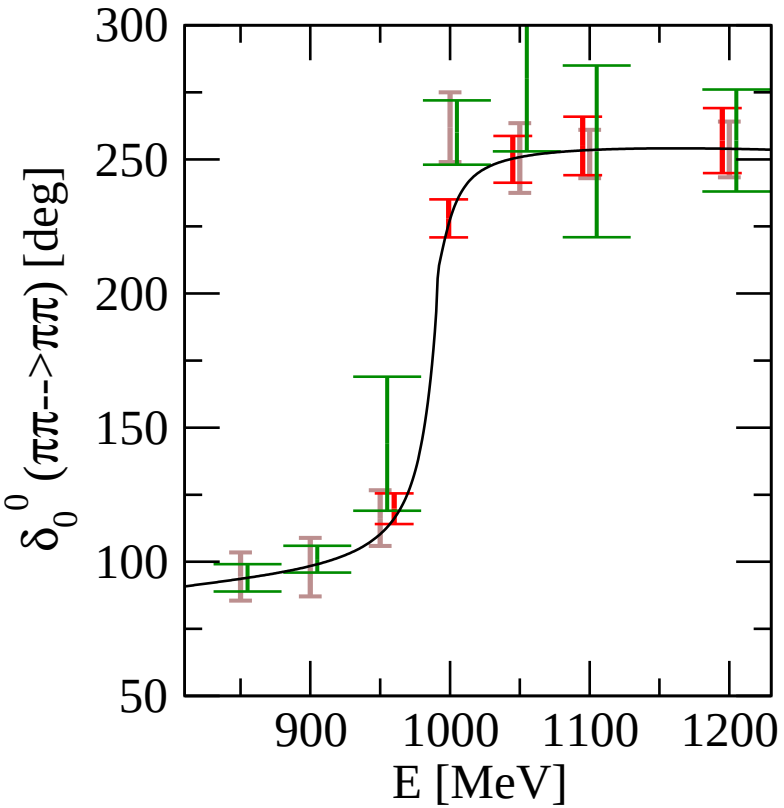




Multi-channel dynamics

Error propagation from pseudo lattice-data [M.D., U.-G. Meißner, E. Oset, A. Rusetsky, in prep.]

- Coupled channels $\pi\pi$, $\bar{K}K$: three unknwons
  - $V(\pi\pi \to \pi\pi)$
  - $V(\pi\pi \to \bar{K}K)$
  - $V(\bar{K}K \to \bar{K}K)$
- How good is the reconstructed phase shift using different lattice data?
- Use pseudo-data generated from hadronic model.
- Hadronic input can reduce the error.



- red: twisted boundary conditions
- green: Asymmetric boxes
- brown: different levels

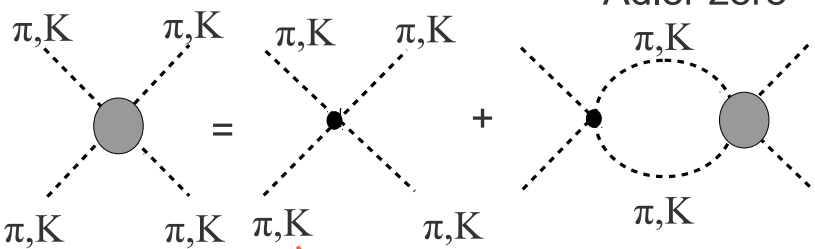
Conclusions

- Lagrangian based, field theoretical description of meson-baryon interaction. Unitarity and analyticity are ensured.
- Meson and baryon exchange are the relevant degrees of freedom in the 2^{nd} and 3^{rd} resonance region. Exchange provides constraints, because all partial waves are linked. With this background, only a few bare states are needed (cleaner resonance extraction).
- $\Rightarrow$ precise determination of model independent resonance parameters (poles).
- Branch points in the complex plane/on the real axis may lead to false resonance signals. Different reaction channels provide more information on resonance content than higher precision in one only.
- Coupled channel formalism links different reactions in one combined description:
  - $\pi N \to \pi N$, $\pi N \to \eta N$, KY,..., $\gamma N \to \pi N$,....
- Dynamical coupled channel models on a momentum lattice (predict levels, error propagation, analyse coming lattice data).

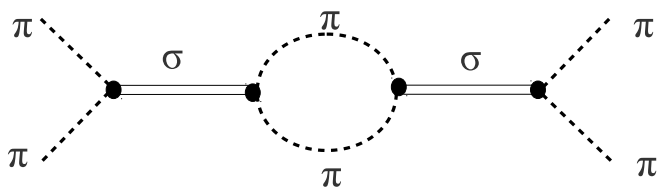
Chiral unitary approach to $\pi\pi$ scattering

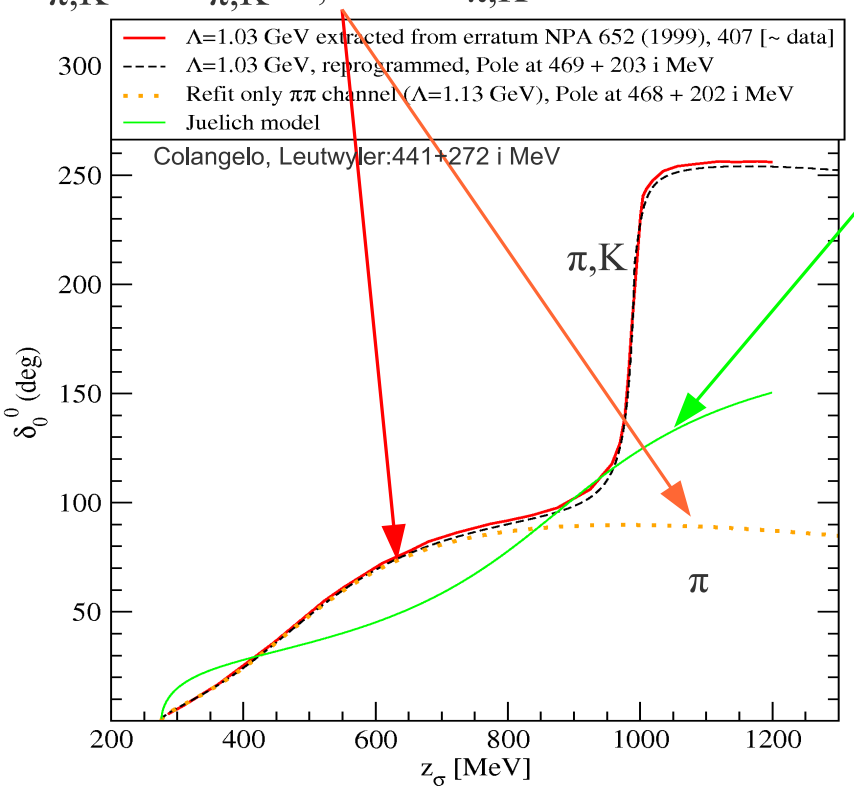
$T_{\pi\pi}=V\chi + V\chi\, G\, T_{\pi\pi}$; $V\chi=(1/2m_\pi^2 - s)/f^2$

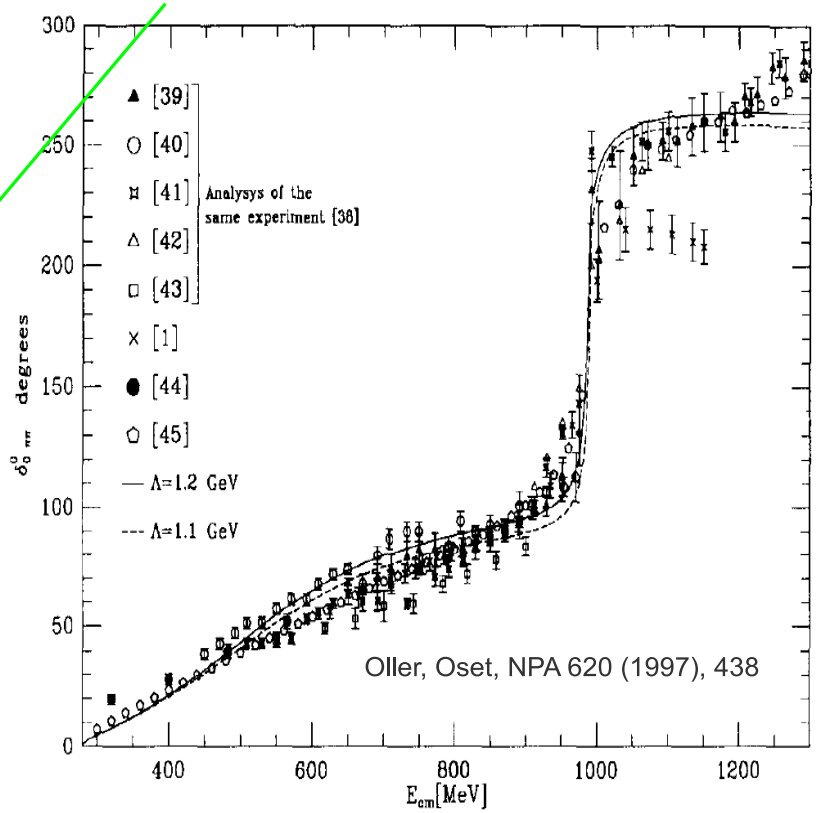
Adler zero



$T_{\pi\pi}=(V_{\sigma\pi\pi})^2 / (z-M-\Sigma_{\pi\pi})$

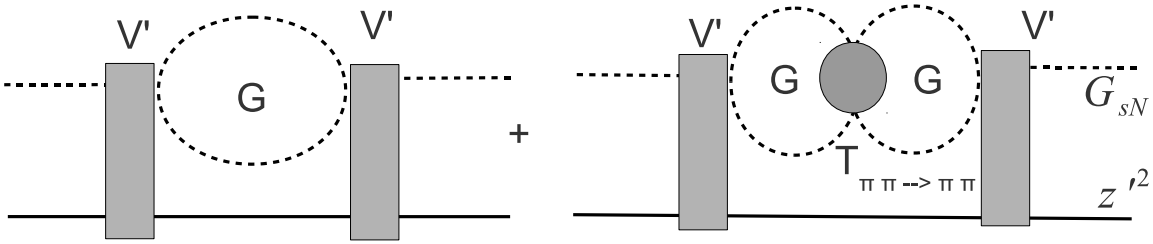






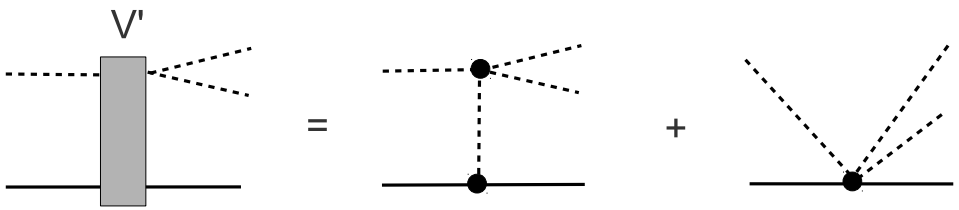
Implementation of the chiral σ

back

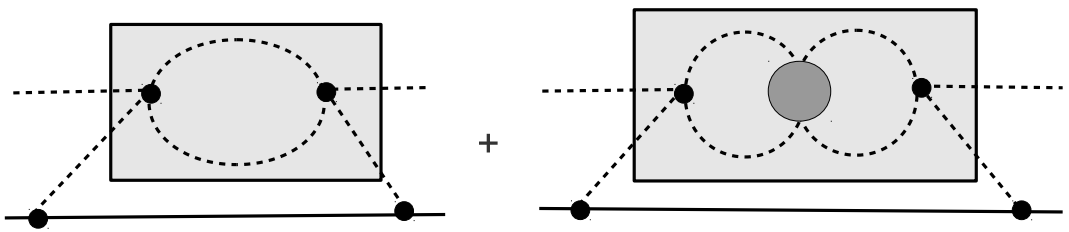


$$G_{sN} = c(z, p)\frac{G(z')}{1 - V(z')G(z')}$$

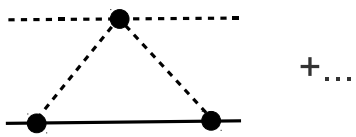
$$z'^2 = z^2 + m_N^2 - 2z\sqrt{p^2 + m_N^2}$$



Independence on parameterization of the pion fields (off-shell cancellatio



ππ scattering as subprocess w/o tree level term



Tree level: Not included (double counting with ρ, σ t-channel exchanges)

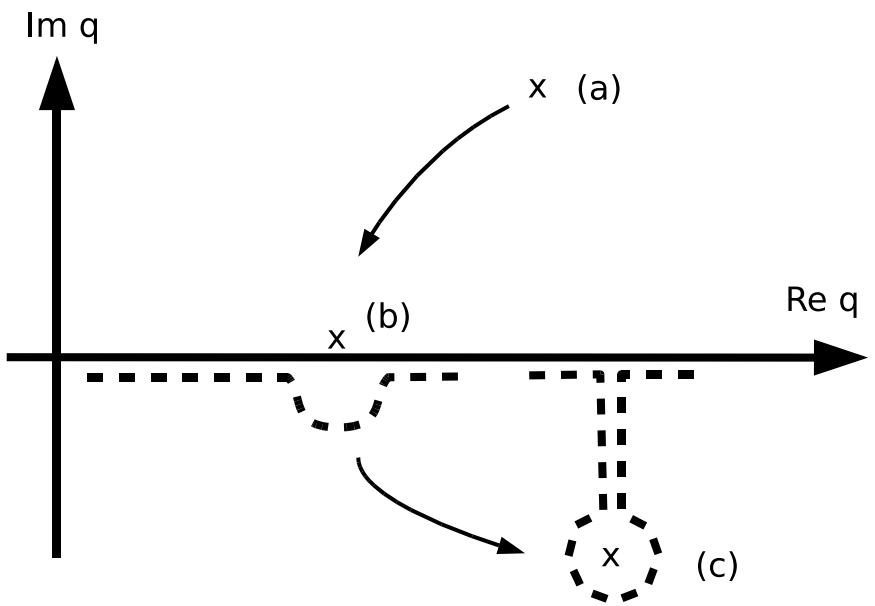
Analytic continuation via Contour deformation

...enables access to all Riemann sheets

$$\Pi_\sigma(z) = \int_0^\infty q^2 dq \, \frac{(v^{\sigma\pi\pi}(q))^2}{z - E_1 - E_2 + i\epsilon}$$

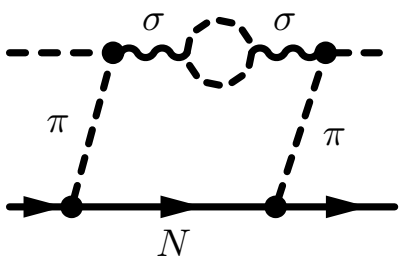
$$z - E_1 - E_2 = 0 \Leftrightarrow q = q_{c.m.}$$

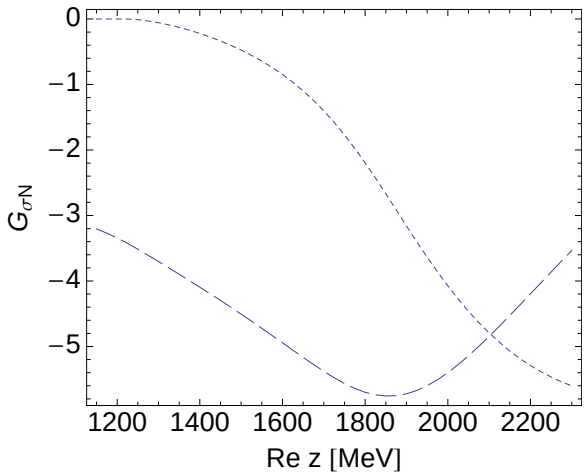
$$q_{c.m.} = \frac{1}{2z}\sqrt{[z^2 - (m_1 - m_2)^2][z^2 - (m_1 + m_2)^2]}$$

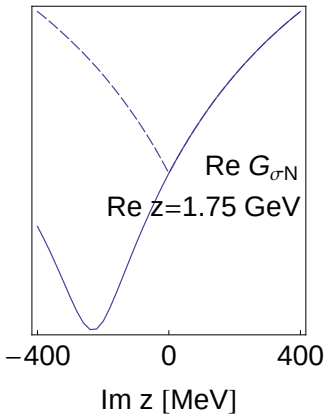


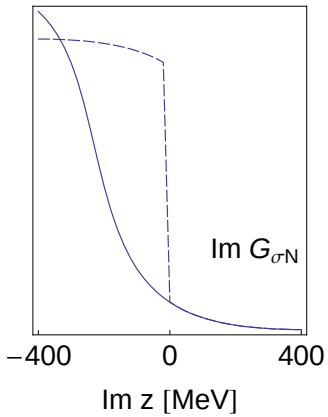
- Plot $q_{c.m.}(z)$ in the q plane of integration (X: Pole positions).
- case (a), Im $z > 0$: straight integration from $q = 0$ to $q = \infty$.
- case (b), Im $z = 0$: Pole is on real q axis.
- case (c), Im $z < 0$: Deformation gives analytic continuation.
- Special case: Pole at $q = 0$ $\Leftrightarrow$ branch point at $z = m_1 + m_2$ ($=$ threshold).

Propagator of effective $\pi\pi N$ channels σN, ρN, $\pi\Delta$





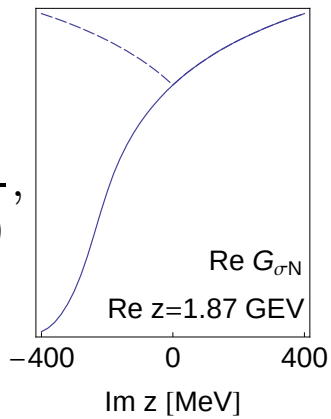


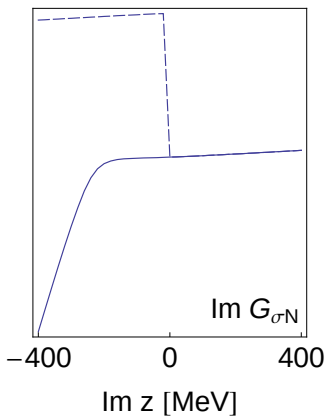


$$g_{\sigma N}(z,k) = \frac{1}{z - \sqrt{m_N^2 + k^2} - \sqrt{(m_\sigma^0)^2 + k^2} - \Pi_\sigma(z_\sigma(z,k),k)},$$

$$G_{\sigma N}(z) = \int_0^\infty dk\, k^2\, F(k)\, g_{\sigma N}(z,k),$$

$$z_\sigma(z,k) = z + m_\sigma^0 - \sqrt{k^2 + (m_\sigma^0)^2} - \sqrt{k^2 + m_N^2}$$

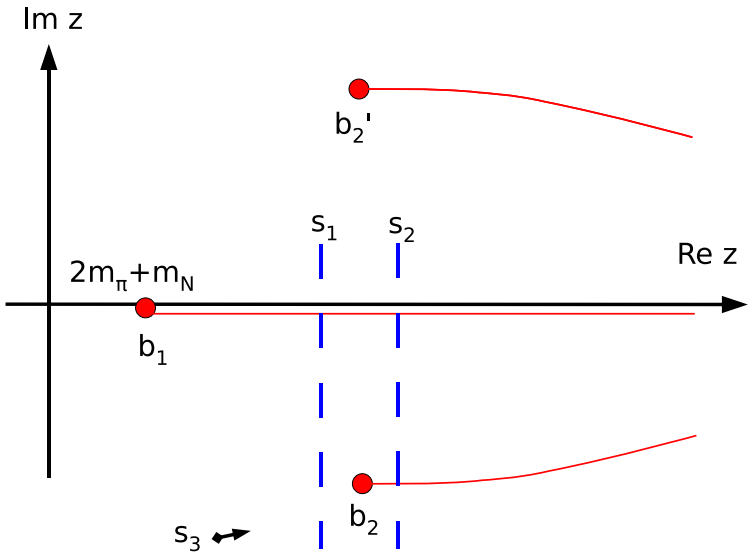




→ Branch point in the complex z plane.

Effective $\pi\pi N$ channels: Analytic structure

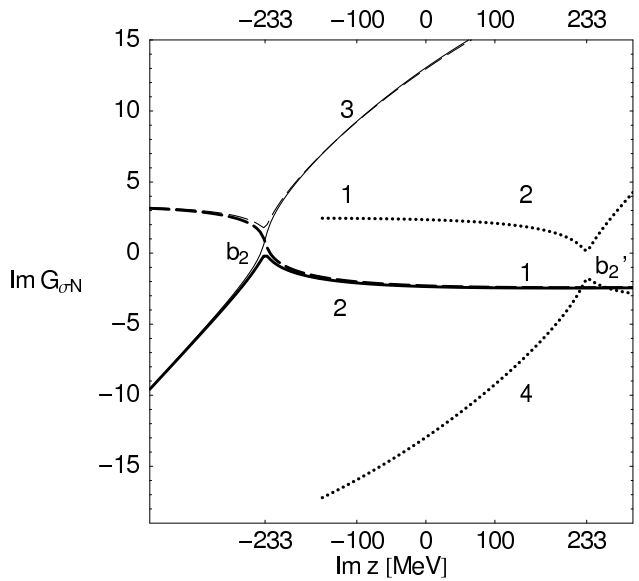
back



- The cut along Im $z = 0$ is induced by the cut of the self energy of the unstable particle.
- The poles of the unstable particle (σ) induce branch points in the σN propagator at

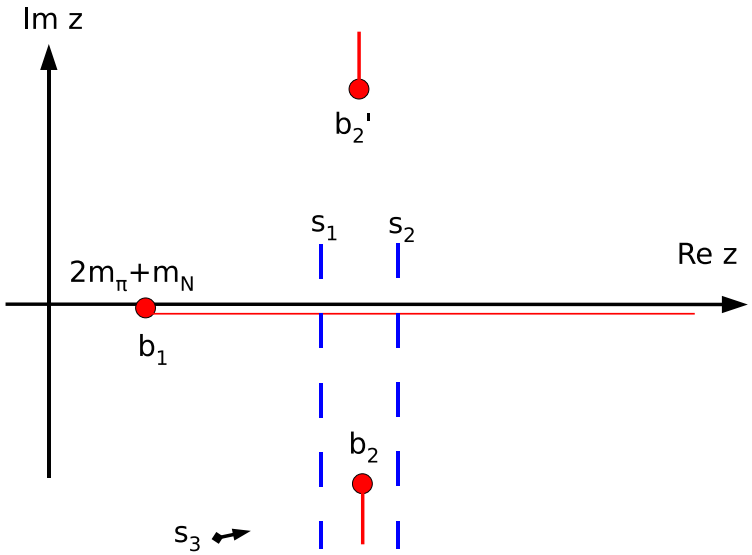
$$z_{b_2} = m_N + z_0, \; z_{b_2'} = m_N + z_0^*$$

Three branch points and four sheets for each of the σN, ρN, and $\pi\Delta$ propagators.



Effective $\pi\pi N$ channels: Analytic structure

back



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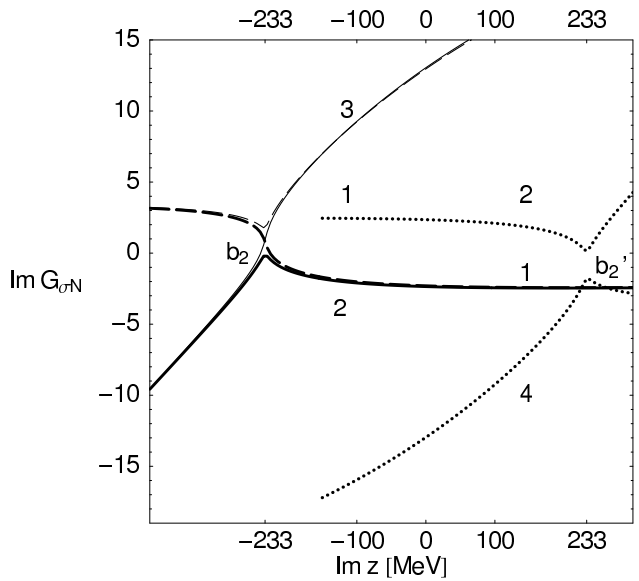

Branch points in coupled channels ($\gamma N \to \eta N$)

[M.D., K. Nakayama, EPJA43 (2010), PLB683 (2010)]

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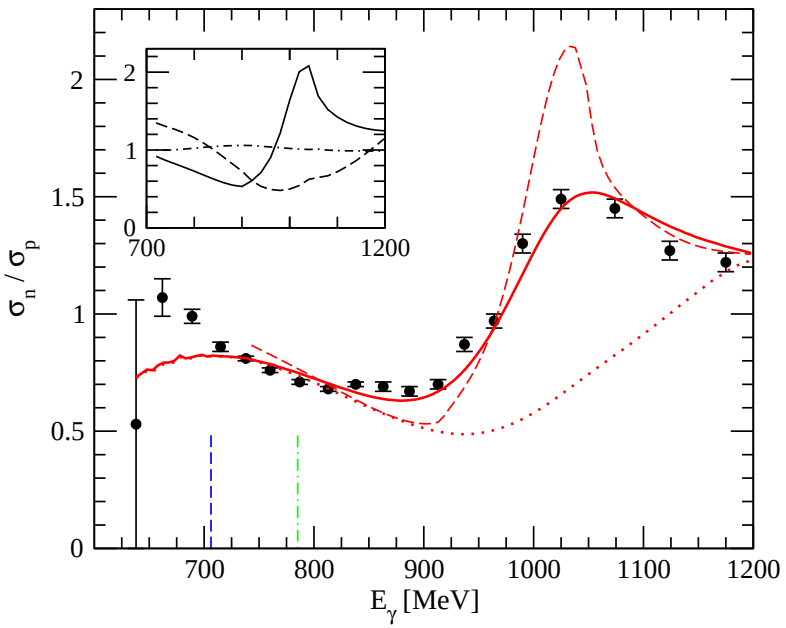
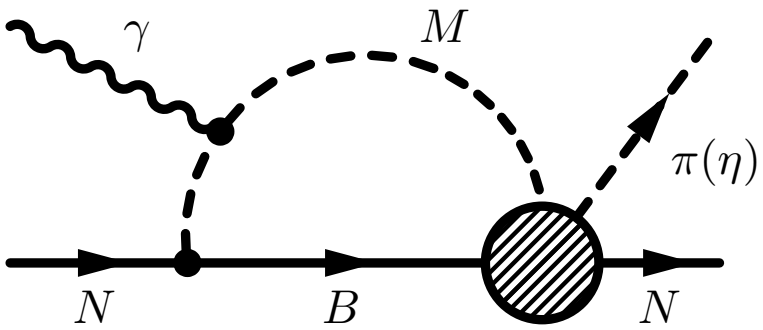


[Data: I. Jaegle et al., CBELSA/TAPS, PRL 100 (2008)]



- $MB = \pi N,\ \eta N, K\Lambda, K\Sigma$
- Pronounced cusp from dispersive ("real") part of the loop.
- Analyticity is crucial.

Branch points in coupled channels ($\gamma N \to \eta N$)

[M.D., K. Nakayama, EPJA43 (2010), PLB683 (2010)]

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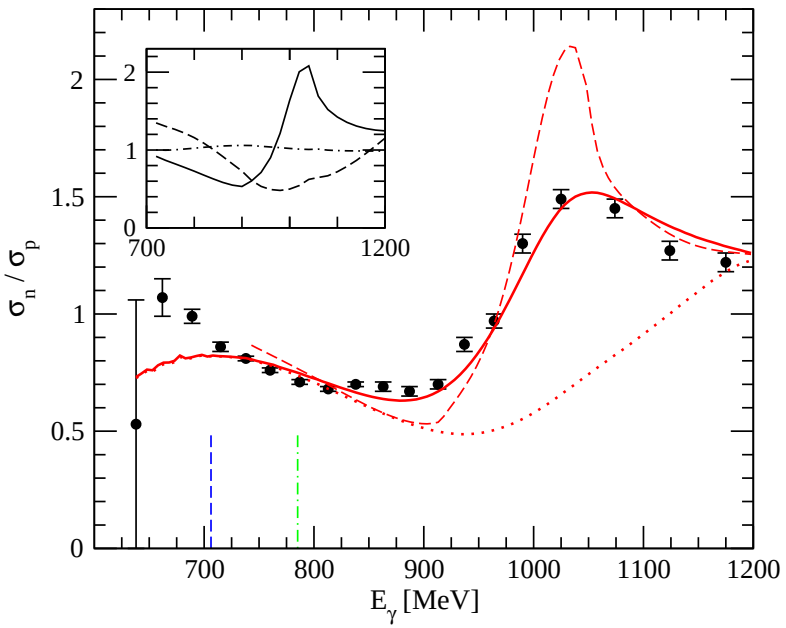
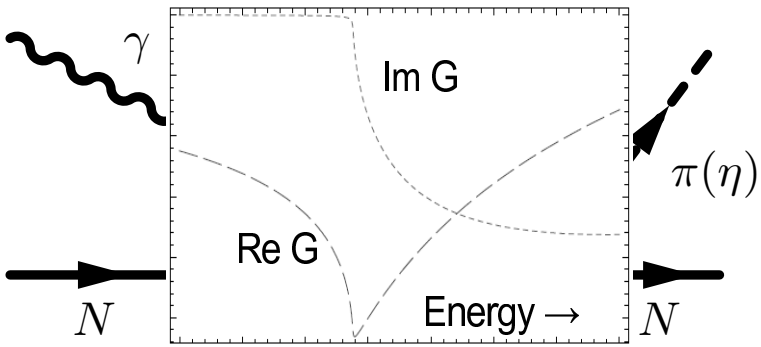


[Data: I. Jaegle et al., CBELSA/TAPS, PRL 100 (2008)]



- $MB = \pi N,\ \eta N,\ K\Lambda,\ K\Sigma$
- Pronounced cusp from dispersive ("real") part of the loop.
- Analyticity is crucial.

Couplings " $g = \sqrt{a_{-1}}$ " to other channels

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	$N\pi$	$N\rho^{(1)}$ $(S=1/2)$	$N\rho^{(2)}$ $(S=3/2)$	$N\rho^{(3)}$ $(S=$
$N^*(1535)\ S_{11}$	$S_{11}\ \ 8.1+0.5i$	$S_{11}\ \ 2.2-5.4i$	–	$D_{11}\ \ 0.5$
$N^*(1650)\ S_{11}$	$S_{11}\ \ 8.6-2.8i$	$S_{11}\ \ 0.9-9.1i$	–	$D_{11}\ \ 0.3$
$N^*(1440)\ P_{11}$	$P_{11}\ 11.2-5.0i$	$P_{11}\ -1.3+3.2i$	$P_{11}\ \ 3.6-2.6i$	–
$\Delta^*(1620)\ S_{31}$	$S_{31}\ \ 2.9-3.7i$	$S_{31}\ \ 0.0-0.0i$	–	$D_{31}\ \ 0.0$
$\Delta^*(1910)\ P_{31}$	$P_{31}\ \ 1.2-3.5i$	$P_{31}\ \ 0.2-0.4i$	$P_{31}\ -0.2-0.4i$	–
$N^*(1720)\ P_{13}$	$P_{13}\ \ 3.7-2.6i$	$P_{13}\ \ 0.1+0.8i$	$P_{13}\ -1.1+0.1i$	$F_{13}\ \ 0.1$
$N^*(1520)\ D_{13}$	$D_{13}\ \ 8.4-0.8i$	$D_{13}\ -0.6+0.7i$	$D_{13}\ \ 0.9-2.0i$	$S_{13}\ -2.5$
$\Delta(1232)\ P_{33}$	$P_{33}\ 17.9-3.2i$	$P_{33}\ -1.3-0.8i$	$P_{33}\ -0.9-3.0i$	$F_{33}\ \ 0.0$
$\Delta^*(1700)\ D_{33}$	$D_{33}\ \ 4.9-1.0i$	$D_{33}\ -0.2+0.9i$	$D_{33}\ -0.4-0.4i$	$S_{33}\ -0.1$

	$N\eta$	$\Delta\pi^{(1)}$	$\Delta\pi^{(2)}$	$N\sigma$
$N^*(1535)\ S_{11}$	$S_{11}\ 11.9-2.3i$	–	$D_{11}\ \ -5.9+4.8i$	$P_{11}\ -1.4$
$N^*(1650)\ S_{11}$	$S_{11}\ -3.0+0.5i$	–	$D_{11}\ \ 4.3+0.4i$	$P_{11}\ -2.1$
$N^*(1440)\ P_{11}$	$P_{11}\ -0.1+0.0i$	$P_{11}\ \ -4.6-1.7i$	–	$S_{11}\ -8.3$
$\Delta^*(1620)\ S_{31}$	–	–	$D_{31}\ \ 11.1-4.0i$	–
$\Delta^*(1910)\ P_{31}$	–	$P_{31}\ \ 15.0-0.3i$	–	–
$N^*(1720)\ P_{13}$	$P_{13}\ -7.7+5.5i$	$P_{13}\ -14.1+3.0i$	$F_{13}\ \ 0.0-0.3i$	$D_{13}\ -0.8$
$N^*(1520)\ D_{13}$	$D_{13}\ \ 0.16-0.60i$	$D_{13}\ \ 0.0+0.4i$	$S_{13}\ -12.9-0.7i$	$P_{13}\ -0.6$
$\Delta(1232)\ P_{33}$	–	$P_{33}\ -(4\text{ to }5)+i(0\text{ to }0.5)$	$F_{33}\ \ \sim 0$	–
$\Delta^*(1700)\ D_{33}$	–	$D_{33}\ \ -0.7-0.3i$	$S_{33}\ -19.7+4.5i$	–

Resonance couplings g_i $[10^{-3}\,\mathrm{MeV}^{-1/2}]$ to the coupled channels i. Also, the LJS type of each coupling is indicated. For the ρN channels, the total spin S is also indicated.

Zeros and branching ratio to πN, ηN

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first sheet		second sheet		[FA02]
P_{11}	$1235 - 0\,i$	S_{11}	$1587 - 45\,i$	$1578 - 38\,i$
D_{33}	$1396 - 78\,i$	S_{31}	$1585 - 17\,i$	$1580 - 36\,i$
		P_{31}	$1848 - 83\,i$	$1826 - 197\,i$
		P_{13}	$1607 - 38\,i$	$1585 - 51\,i$
		P_{33}	$1702 - 64\,i$	–
		D_{13}	$1702 - 64\,i$	$1759 - 64\,i$

Position of zeros of the full amplitude T in [MeV]. [FA02]: Arndt et al., PRC 69 (2004).

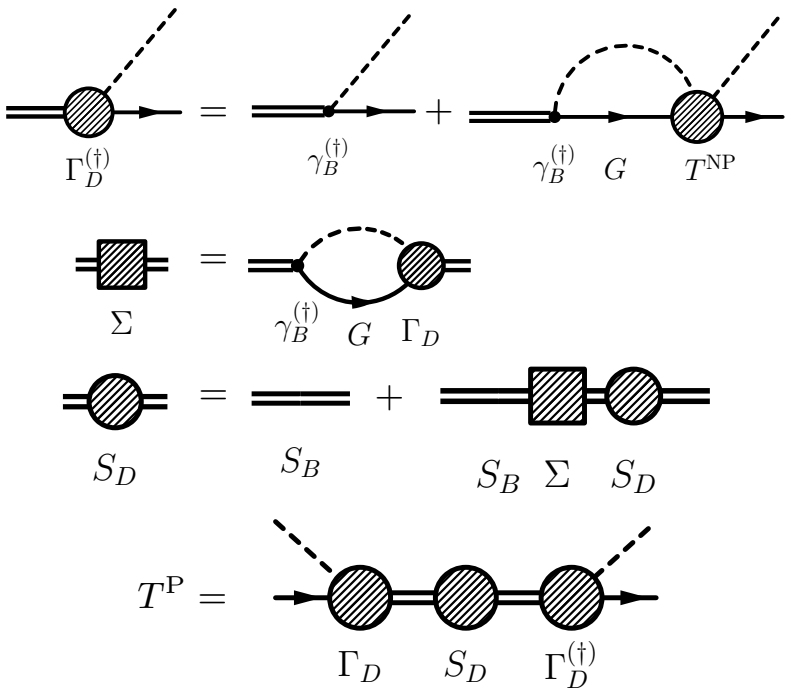
	$\Gamma_{\pi N}/\Gamma_{\rm Tot}$ [%]	$\Gamma_{\eta N}/\Gamma_{\rm Tot}$ [%]
$N^*(1535)\,S_{11}$	48 [33 to 55]	38 [45 to 60]
$N^*(1650)\,S_{11}$	79 [60 to 95]	6 [3 to 10]
$N^*(1440)\,P_{11}$	64 [55 to 75]	0 [0 ± 1]
$\Delta^*(1620)\,S_{31}$	34 [20 to 30]	–
$\Delta^*(1910)\,P_{31}$	11 [15 to 30]	–
$N^*(1720)\,P_{13}$	13 [10 to 20]	38 [4 ± 1]
$N^*(1520)\,D_{13}$	67 [55 to 65]	0.10 [0.23 ± 0.04]
$\Delta(1232)\,P_{33}$	100 [100]	–
$\Delta^*(1700)\,D_{33}$	13 [10 to 20]	–

Branching ratios into πN and ηN. The values in brackets are from the PDG, [Amsler et al., PLB 667 (2008)].

Couplings and dressed vertices

Residue a_{-1} vs. dressed vertex Γ vs. bare vertex γ.

back



$$
\begin{aligned}
a_{-1} &= \frac{\Gamma_d \, \Gamma_d^{(\dagger)}}{1 - \frac{\partial}{\partial Z}\Sigma} \\
g &= \sqrt{a_{-1}} \\
r &= |(\Gamma_D - \gamma_B)/\Gamma_D|, \\
r' &= |1 - \sqrt{1 - \Sigma'}|,
\end{aligned}
$$

- Dressed Γ depends on $T^{\rm NP}$.
- $\boxed{\sqrt{a_{-1}} \neq \Gamma \neq \gamma}$

	γ^C	Γ^C	r [%]	r' [%]
$N^*(1520)\ D_{13}$	$6.4 - 0.6i$	$13.2 + 1.2i$	53	61
$N^*(1720)\ P_{13}$	$-0.1 + 5.4i$	$0.9 + 4.8i$	24	45
$\Delta(1232)\ P_{33}$	$1.3 + 13.0i$	$-2.8 + 22.2i$	45	40
$\Delta^*(1620)\ S_{31}$	$0.1 + 14.3i$	$5.0 + 5.7i$	130	66
$\Delta^*(1700)\ D_{33}$	$5.4 - 0.8i$	$6.7 + 1.0i$	33	54
$\Delta^*(1910)\ P_{31}$	$9.4 + 0.3i$	$1.9 - 3.2i$	222	22

Pole repulsion in P_{33}

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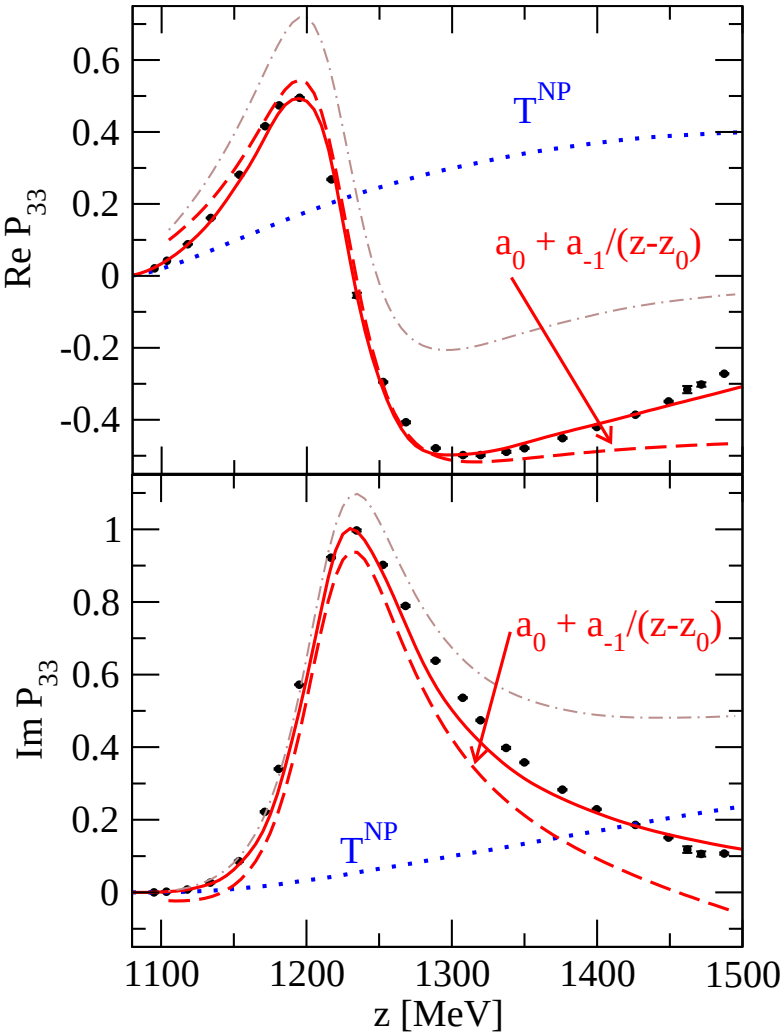
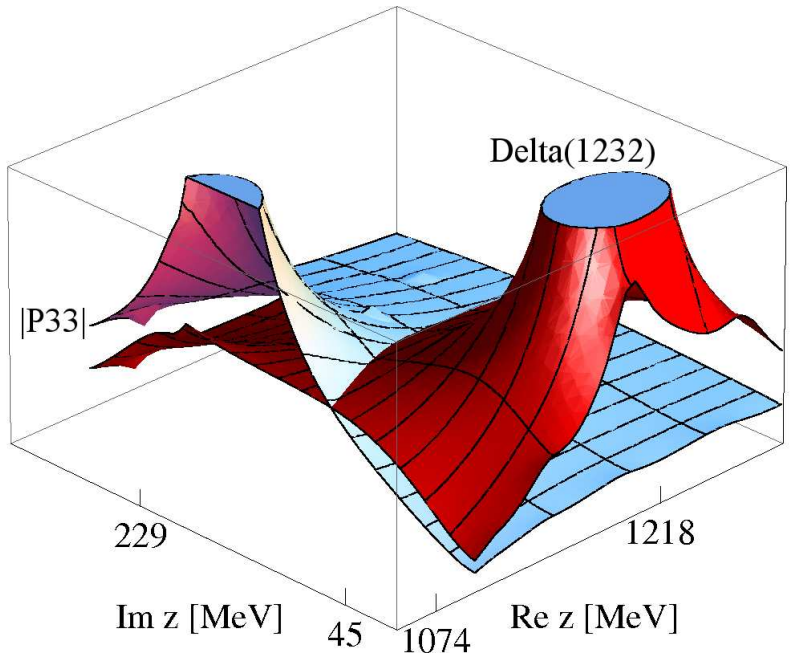


- Poles in T^{NP} may occur $\Rightarrow$ pole repulsion in $T = T^{\mathrm{NP}} + T^{\mathrm{P}}$!



g_{fi} und h_{fi} in JLS-Basis:

$$
\begin{aligned}
g_{fi} \;=\; & \frac{1}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{\frac{1}{2}\,\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\,\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\,\frac{1}{2}}\right] \cos\frac{\theta}{2} \\
& + \frac{1}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{-\frac{1}{2}\,\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\,\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\,\frac{1}{2}}\right] \sin\frac{\theta}{2}
\end{aligned}
$$

$$
\begin{aligned}
h_{fi} \;=\; & \frac{-i}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{\frac{1}{2}\,\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\,\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\,\frac{1}{2}}\right] \sin\frac{\theta}{2} \\
& + \frac{i}{2\sqrt{k_f k_i}} \sum_j (2j+1)\, d^j_{-\frac{1}{2}\,\frac{1}{2}}(\theta) \left[\tau^{j(j-\frac{1}{2})\,\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\,\frac{1}{2}}\right] \cos\frac{\theta}{2}
\end{aligned}
$$

$$\begin{aligned}
\frac{d\sigma}{d\Omega} &= \frac{k_f}{k_i}(|g_{fi}|^2 + |h_{fi}|^2) \\
&= \frac{1}{2k_i^2}\frac{1}{2}\cdot\left(\left|\sum_j (2j+1)(\tau^{j(j-\frac{1}{2})\frac{1}{2}} + \tau^{j(j+\frac{1}{2})\frac{1}{2}})\cdot d^j_{\frac{1}{2}\frac{1}{2}}(\Theta)\right|^2\right. \\
&\quad \left. + \left|\sum_j (2j+1)(\tau^{j(j-\frac{1}{2})\frac{1}{2}} - \tau^{j(j+\frac{1}{2})\frac{1}{2}})\cdot d^j_{-\frac{1}{2}\frac{1}{2}}(\Theta)\right|^2\right)
\end{aligned}$$

$$\vec{P}_f = \frac{2Re(g_{fi}h^*_{fi})}{|g_{fi}|^2 + |h_{fi}|^2}\cdot\hat{n}$$

$$\beta = \arctan\left(\frac{2Im(h^*_{fi}g_{fi})}{|g_{fi}|^2 - |h_{fi}|^2}\right)$$

Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

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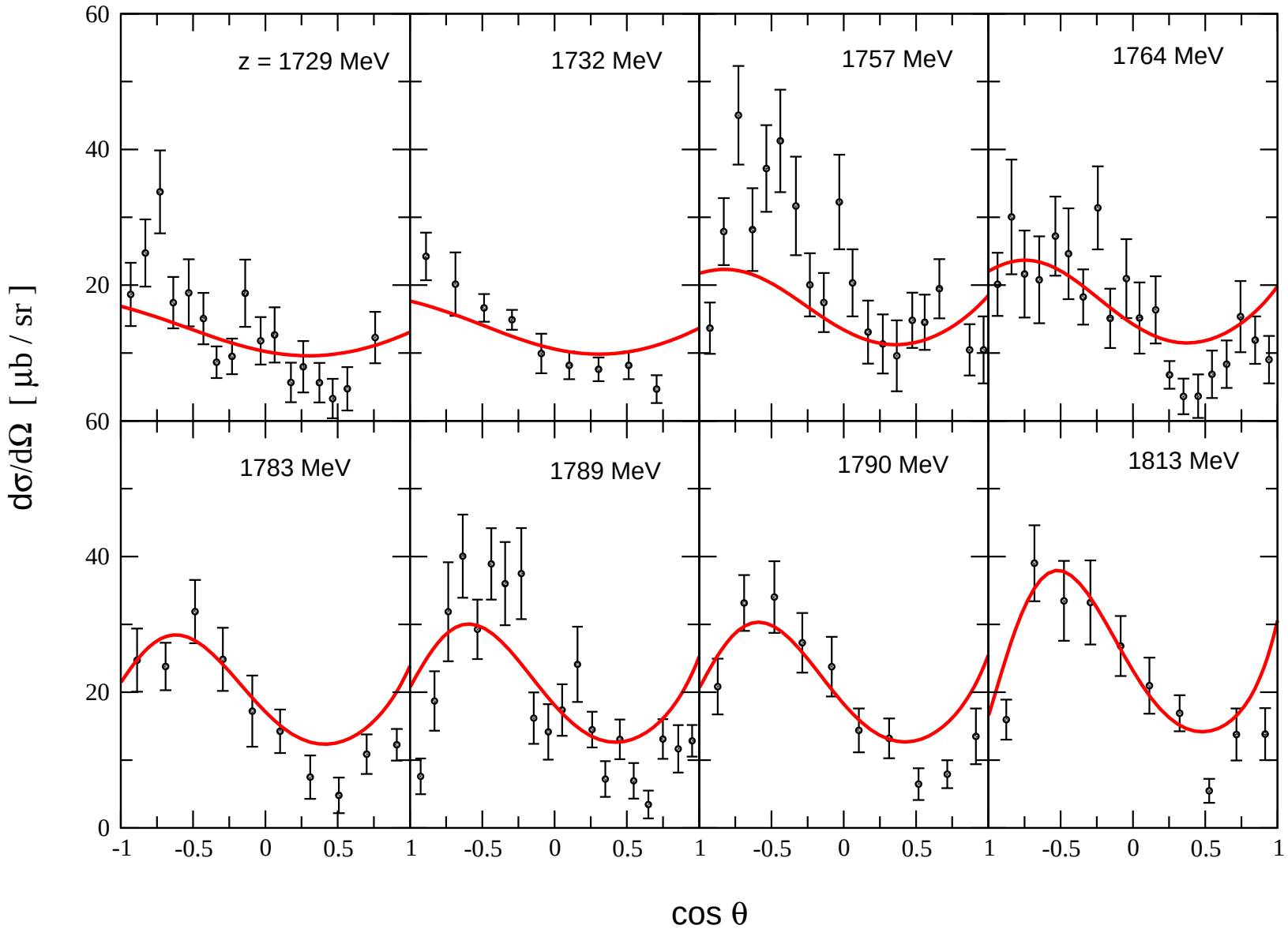
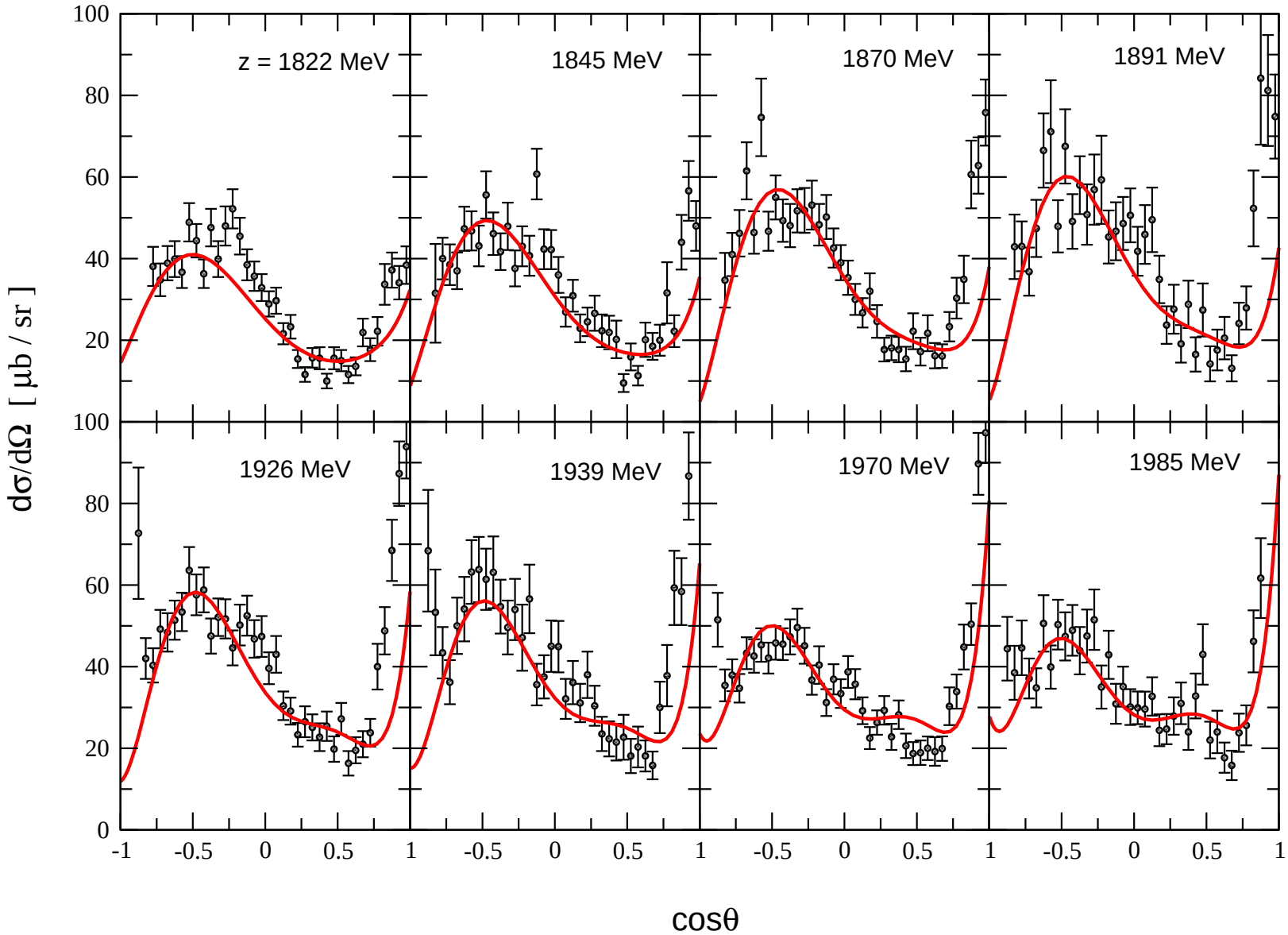


Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

back



Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

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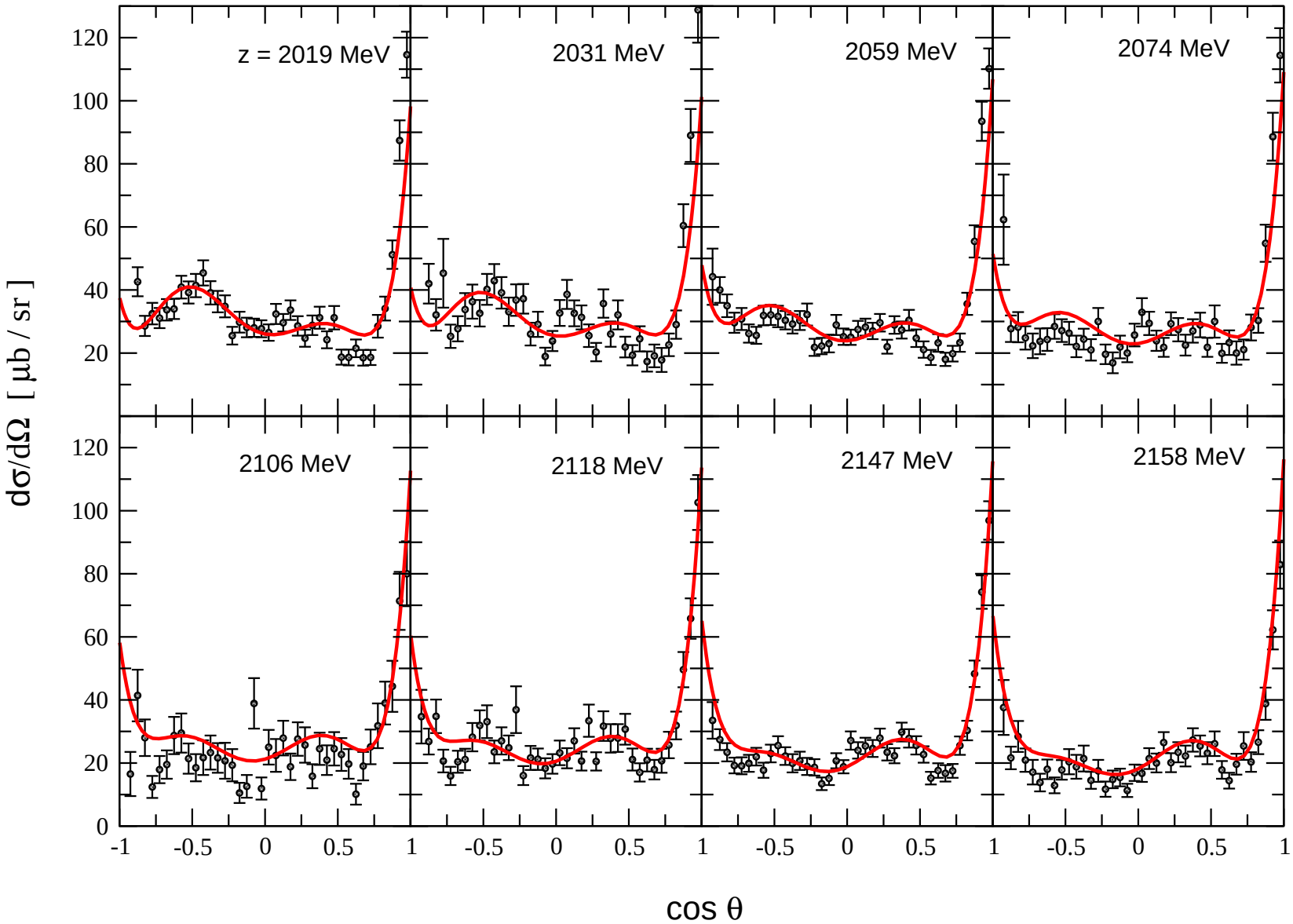
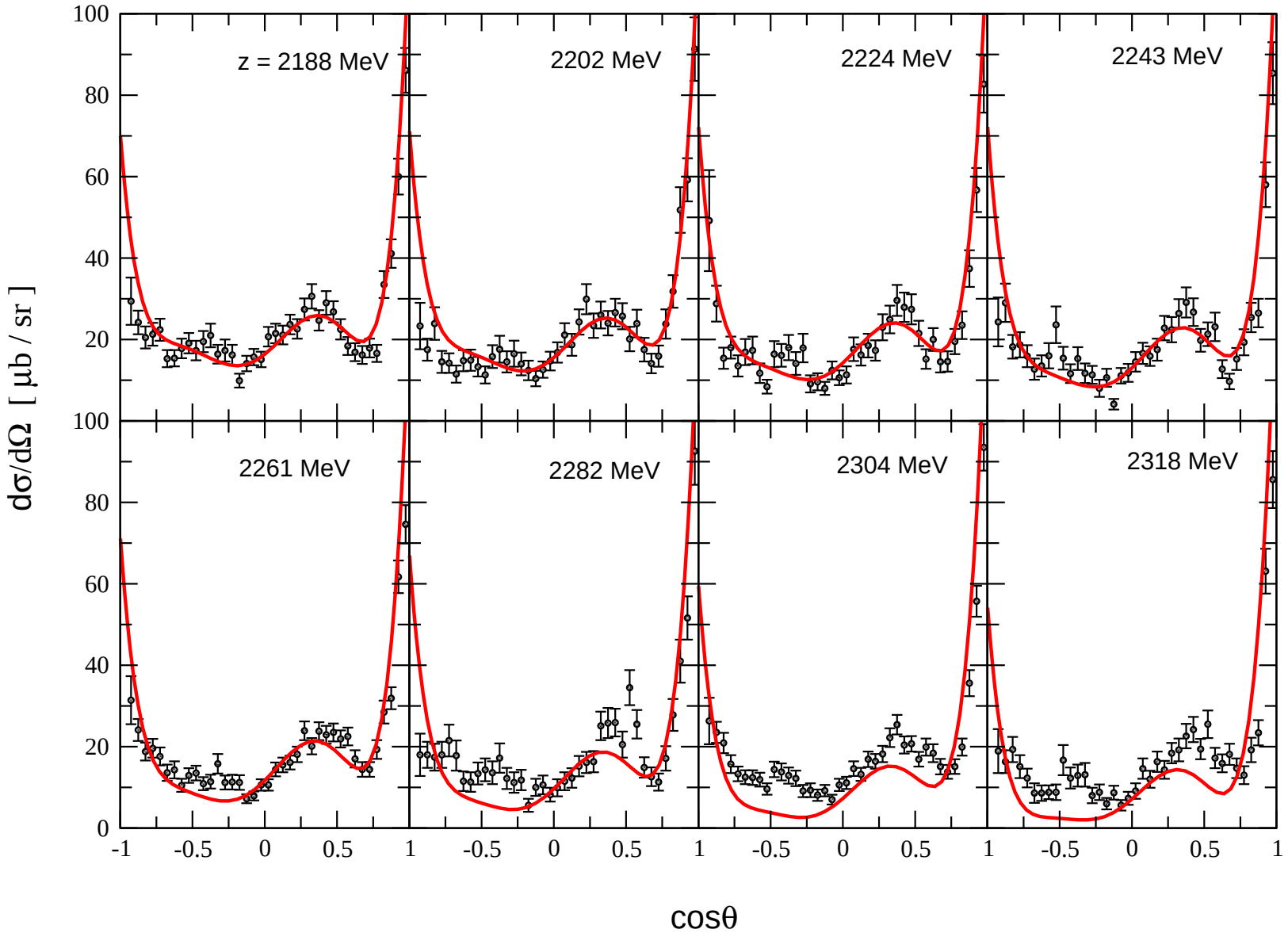
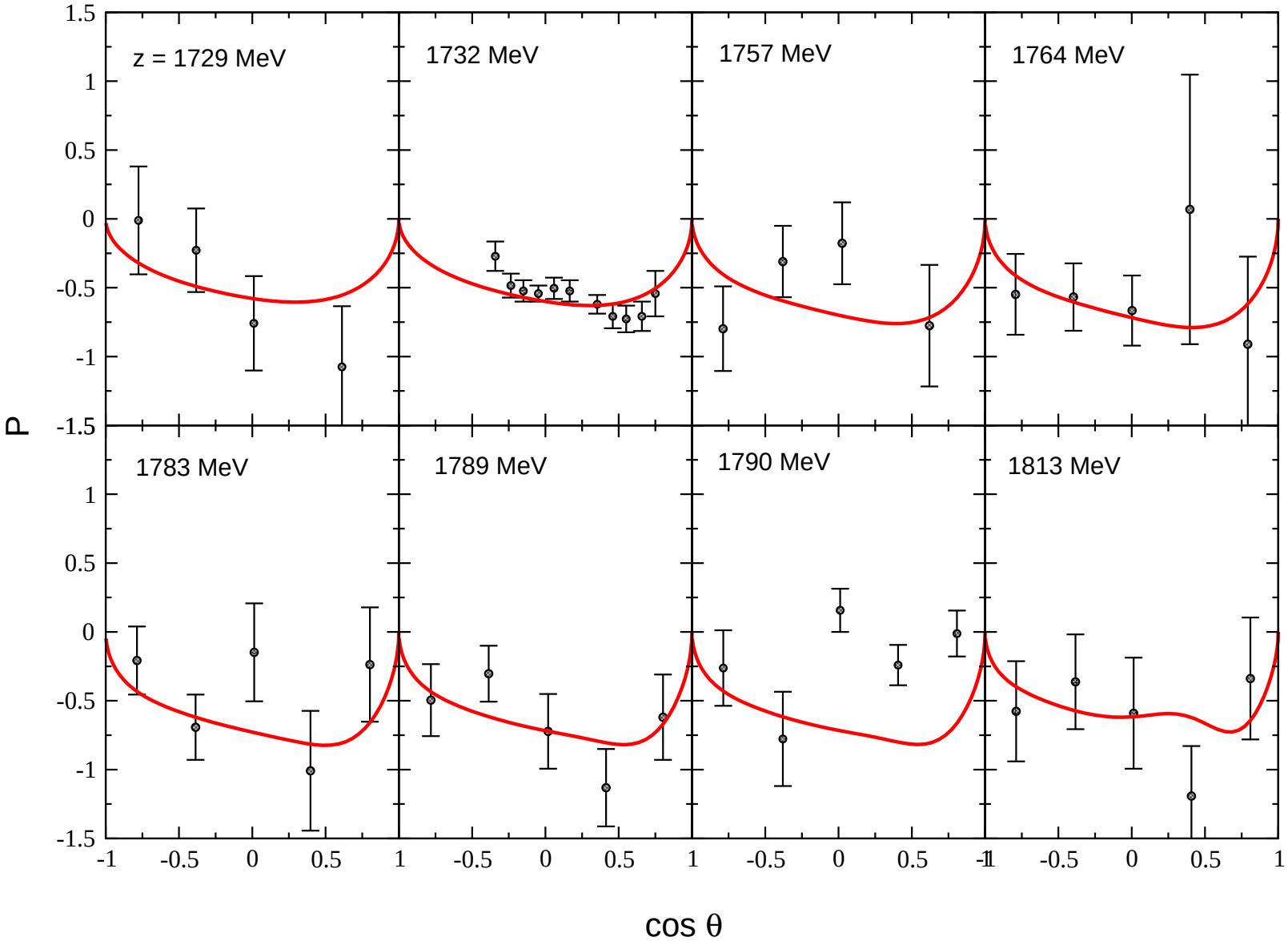


Differential cross section of $\pi^+ p \to K^+ \Sigma^+$

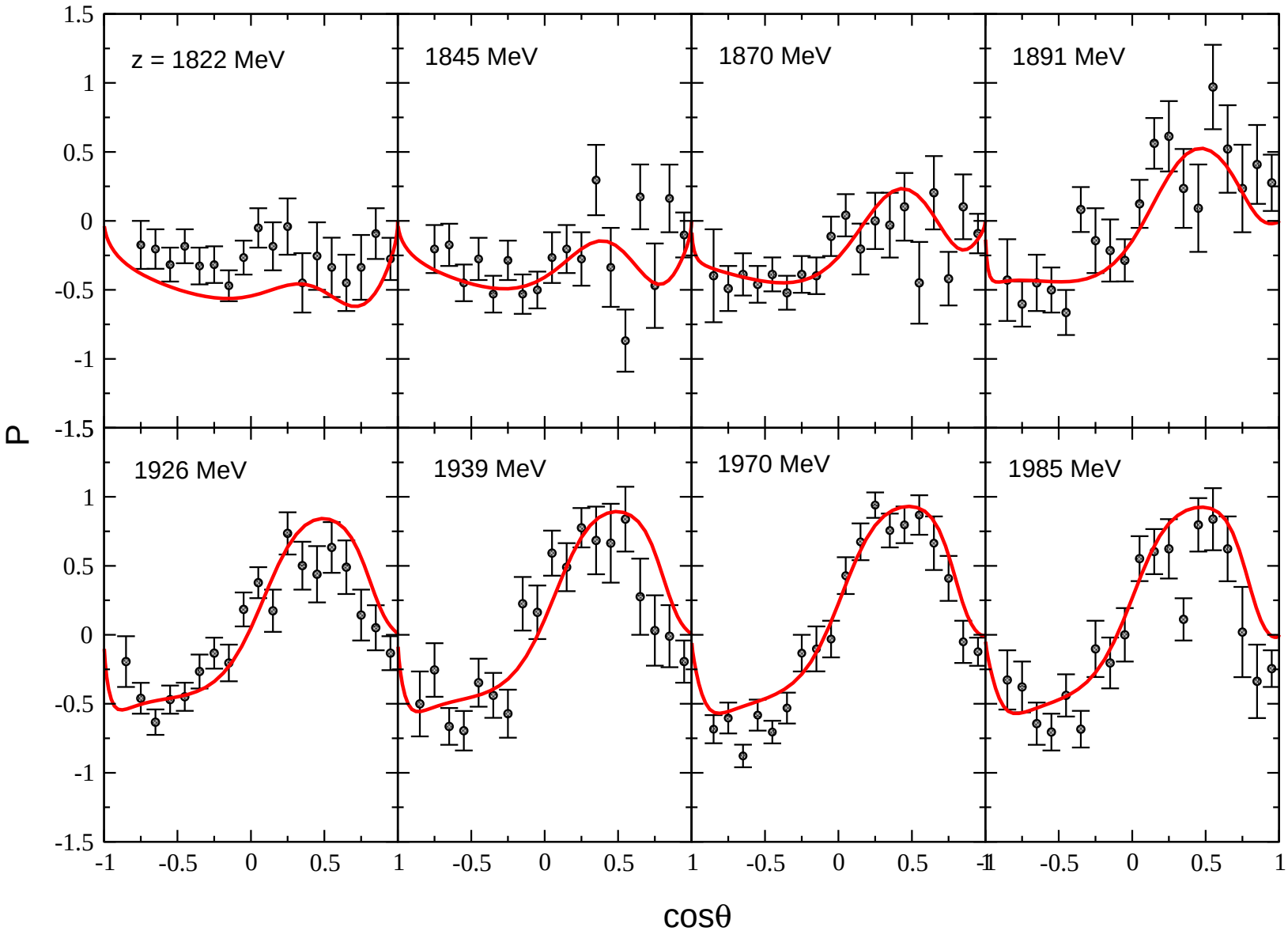
back



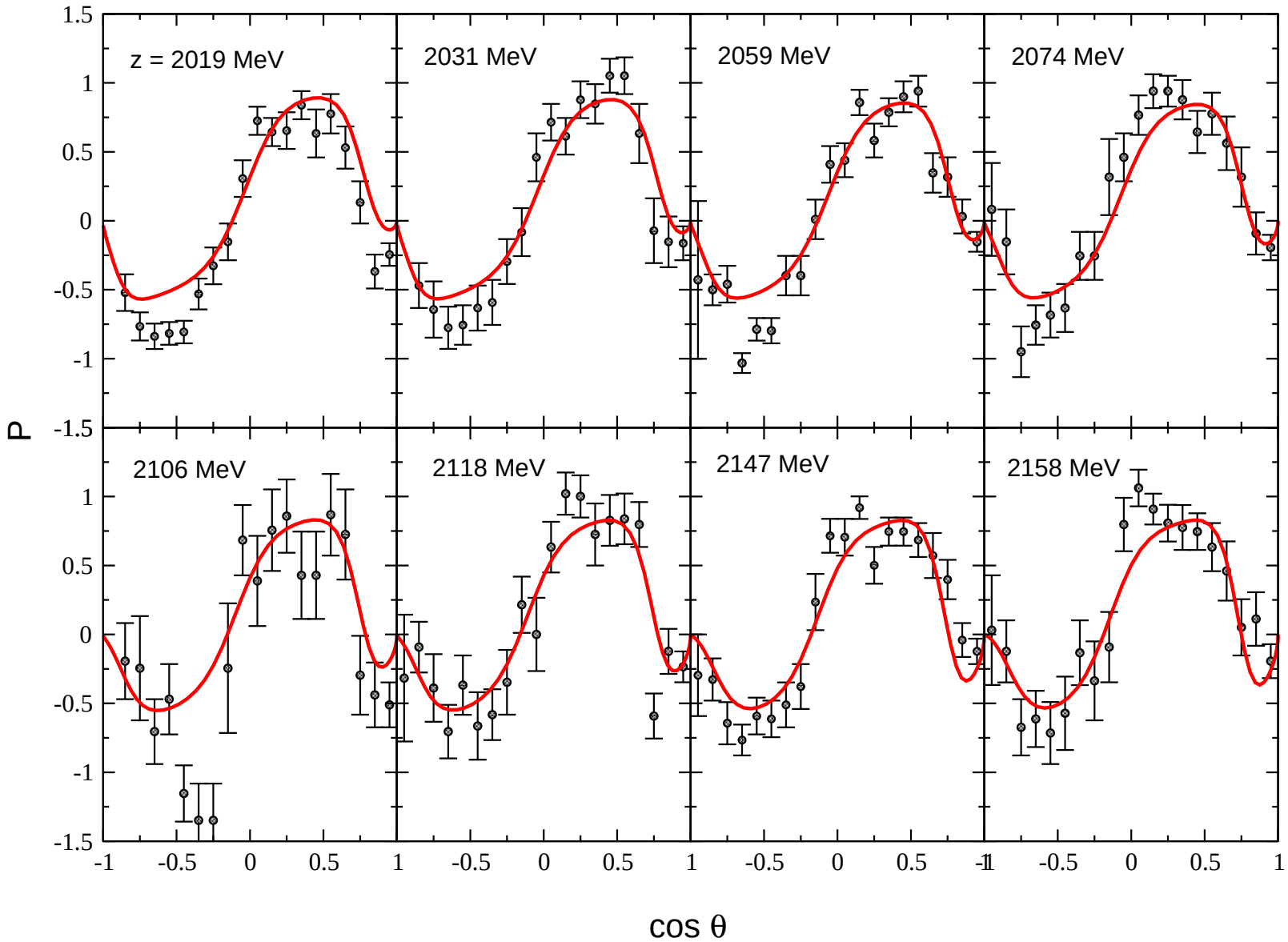
Polarization of $\pi^+ p \to K^+ \Sigma^+$



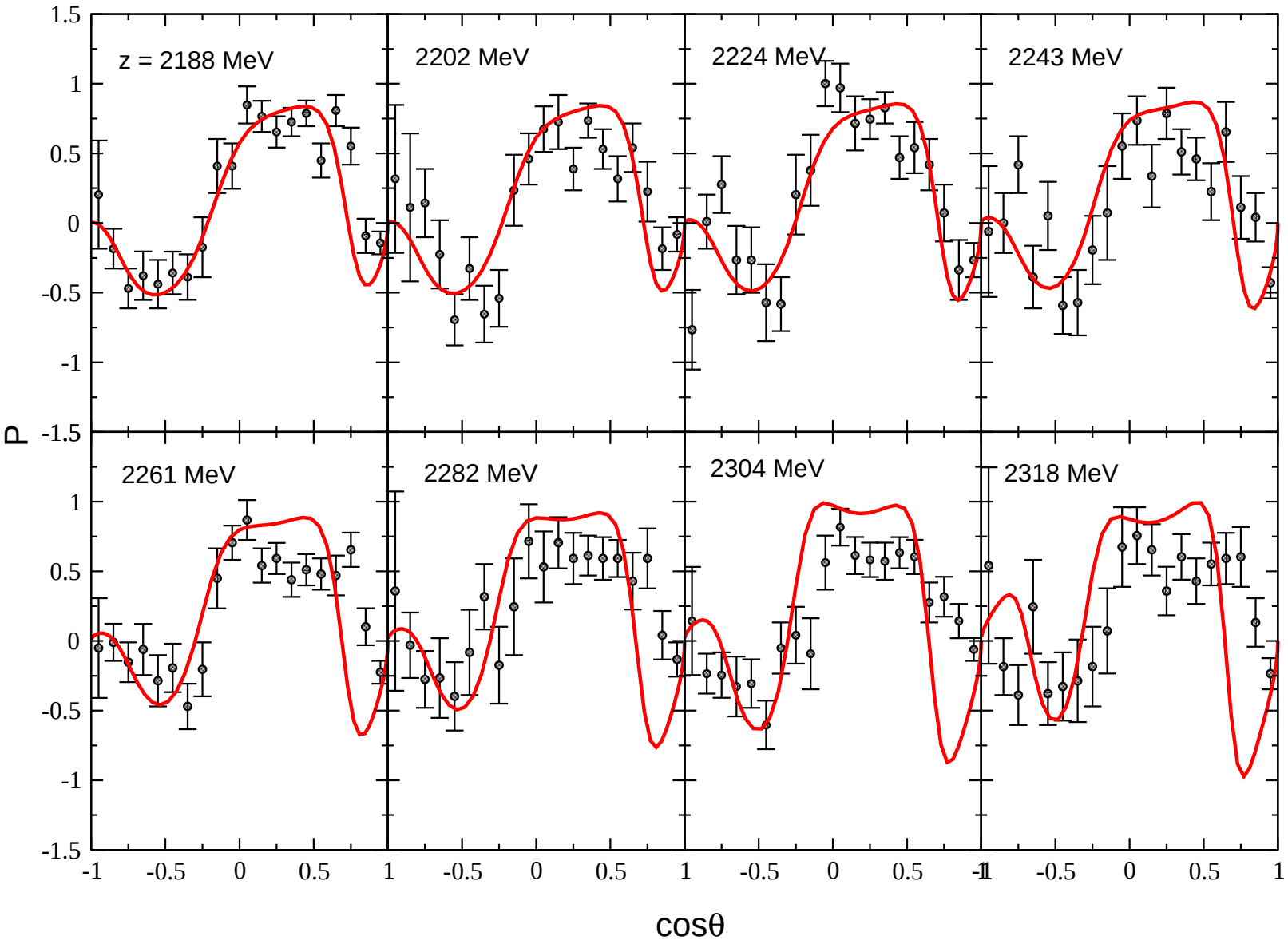
Polarization of $\pi^+ p \to K^+ \Sigma^+$



Polarization of $\pi^+ p \to K^+ \Sigma^+$

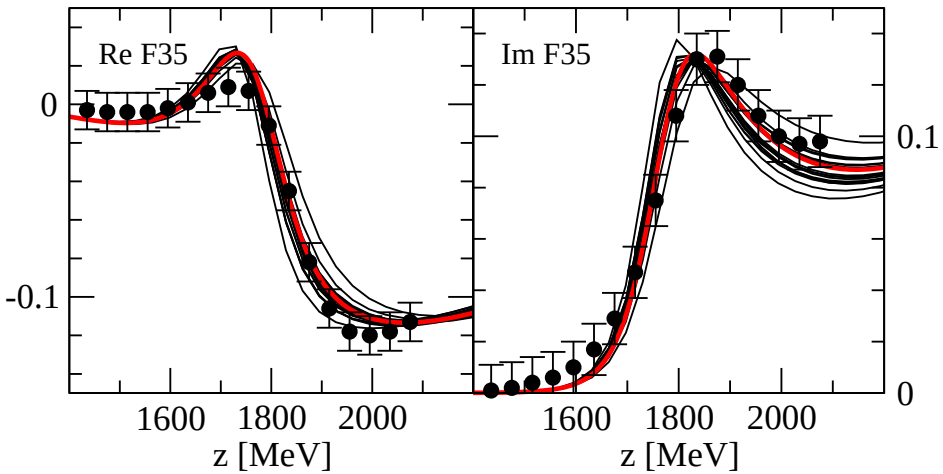


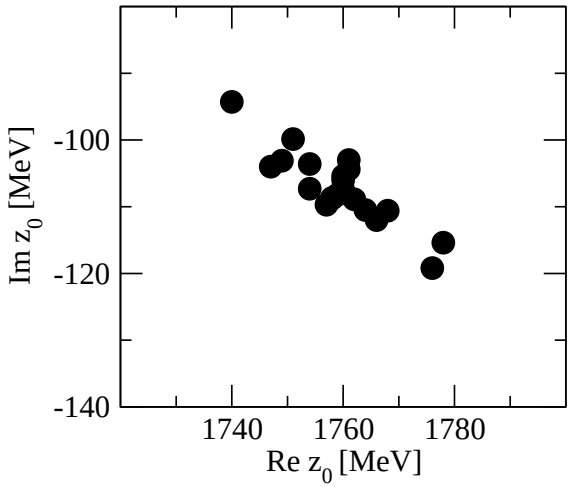
Polarization of $\pi^+ p \to K^+ \Sigma^+$



Error analysis

- Determination of the non-linear parameter error
  - $\chi^2 + 1$ criterion.
  - Varying 39 of 40 parameters to get parameter error.
- Get error on derived quantities like pole positions and residues.
- So far, simplified consideration (error from πN not available, because energy dependent GWU/SAID solution is fitted [PRC74 (2006)]).





Error estimates for parameters and derived quantities

Table: Error estimates of bare mass m_b and bare coupling f for the $\Delta(1905)F_{35}$ resonance.

m_b [MeV]	πN	ρN	$\pi\Delta$	ΣK
2258^{+44}_{-43}	$0.0500^{+0.0011}_{-0.0012}$	$-1.62^{+1.29}_{-1.61}$	$-1.15^{+0.030}_{-0.022}$	$0.120^{+0.0065}_{-0.0059}$

Table: Error estimates of pole position and residues for the $\Delta(1905)F_{35}$ resonance.

			$\pi N \to \pi N$	$\pi N \to K\Sigma$
Re z_0 [MeV]	1764^{+18}_{-20}	$\lvert r\rvert$ [MeV]	$11^{+1.7}_{-1.4}$	$1.4^{+0.24}_{-0.21}$
Im z_0 [MeV]	-109^{+13}_{-12}	θ [^{0}]	$-45^{+3.8}_{-11}$	$-313^{+4.2}_{-10}$

Resonance content from $\pi N \to \pi N$ plus $\pi^+ p \to K^+ \Sigma^+$

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	Re z_0 [MeV] -2 Im z_0 [MeV]	$\lvert r \rvert$ [MeV] θ [^{0}]	$(\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2})/\Gamma_{\rm tot}$ [%] This study	Candlin (1983)	Gießen (2004)
$\Delta(1905)F_{35}$ $5/2^+$ ****	1764 218	1.4 -313	1.23	1.5(3)	$<$1
$\Delta(1910)P_{31}$ $1/2^+$ ****	1721 323	5.5 -6	2.98	$<$3	1.1
$\Delta(1920)P_{33}$ $3/2^+$ ***	1884 229	5.9 -38	5.07	5.2(2)	2.1(3)
$\Delta(1930)D_{35}$ $5/2^-$ ***	1865 147	1.6 -43	2.14	$<$1.5	
$\Delta(1950)F_{37}$ $7/2^+$ ****	1873 206	2.7 -255	2.54	5.3(5)	—

Coupled channels and gauge invariance

Haberzettl, PRC56 (1997), Haberzettl, Nakayama, Krewald, PRC74 (2006),

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Hadronic scattering:

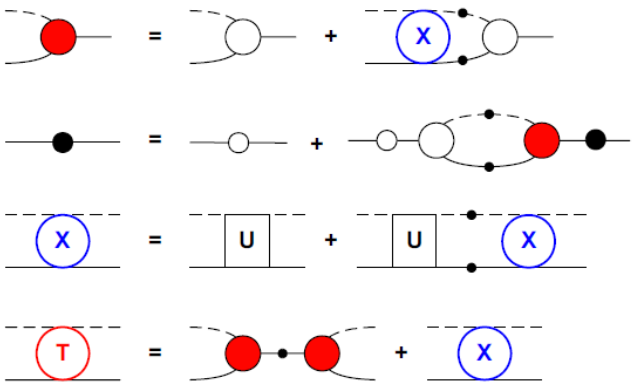
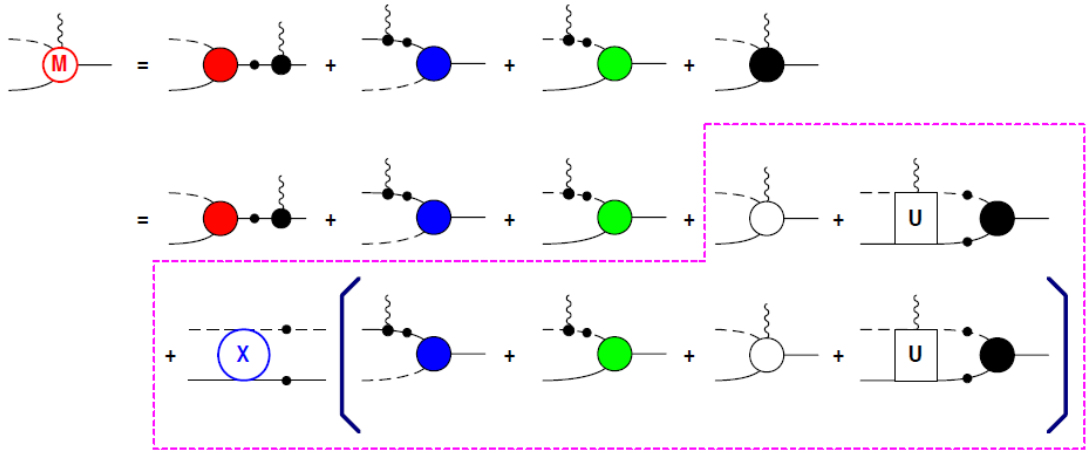


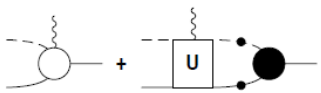
Photo-production:



Gauge invariance: Generalized Ward-Takahashi identity (WTI)

(Note the condition of current conservation $k_\mu M^\mu = 0$ is necessary but not sufficient!)

$$k_\mu M^\mu = -|F_s\tau\rangle S_{p+k} Q_i S_p^{-1} + S_{p'}^{-1} Q_f S_{p'-k} |F_u\tau\rangle + \Delta^{-1}_{p-p'+k} Q_\pi \Delta_{p-p'} |F_t\tau\rangle$$

Strategy: Replace  by phenomenological contact term such that the generalized WTI is satisfied

$d\sigma/d\Omega$ and Σ_γ for $\gamma n \to \pi^- p$

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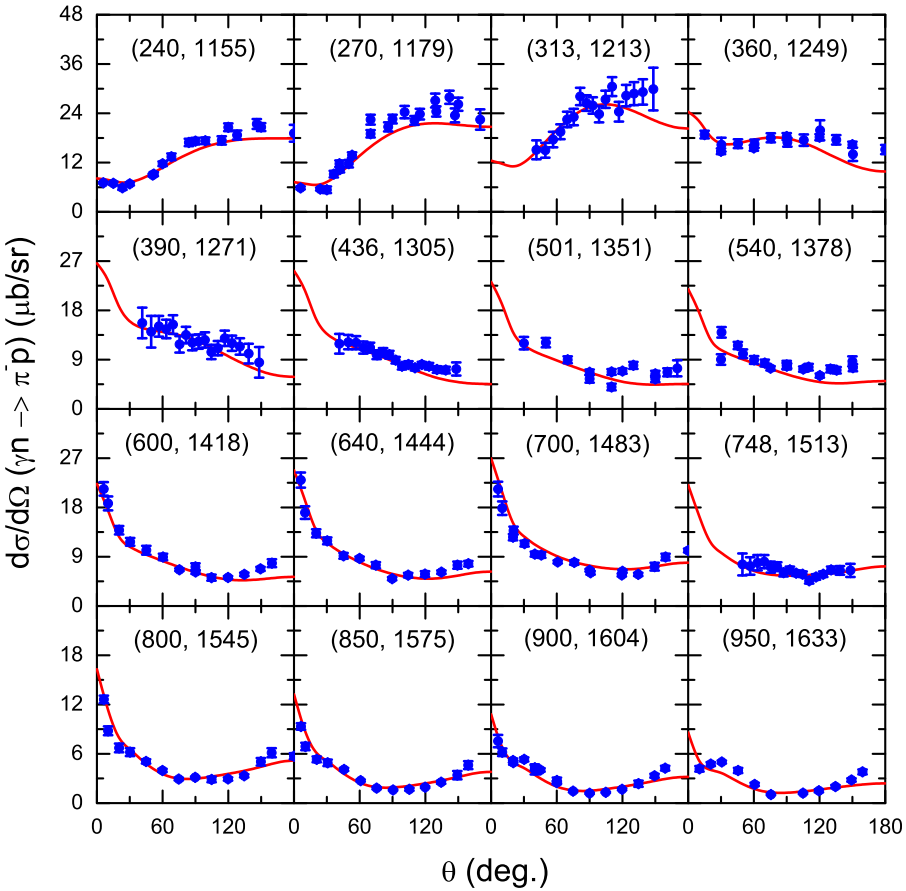
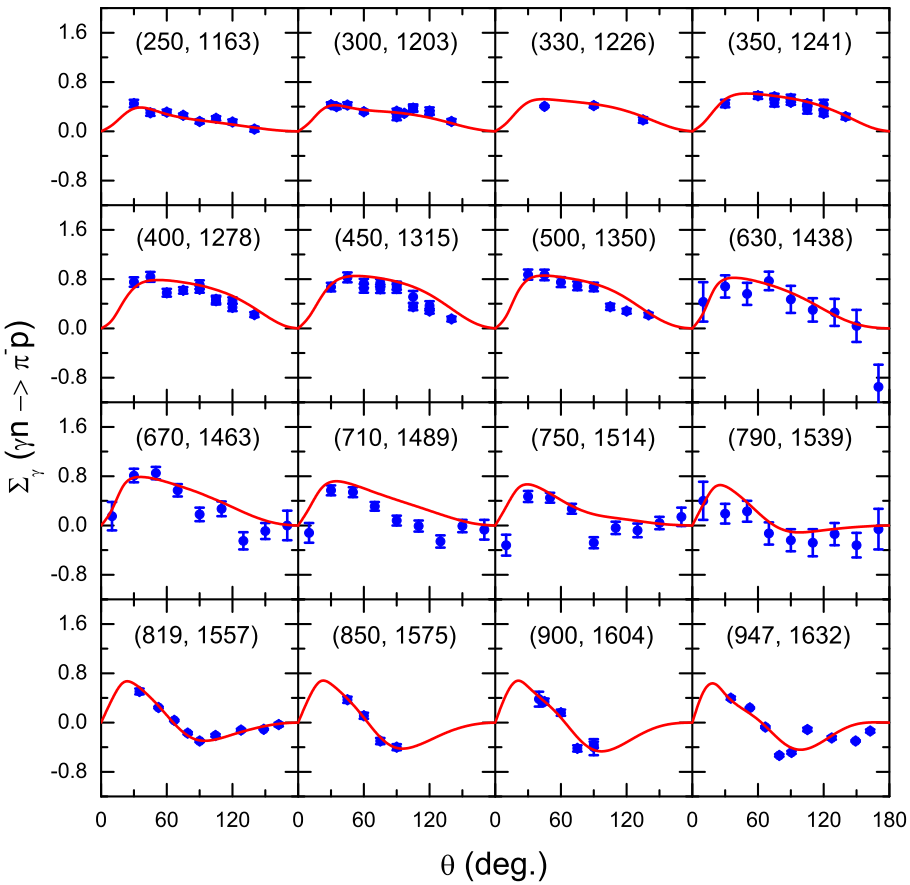


Differential cross section for $\gamma n \to \pi^- p$



Photon spin asymmetry for $\gamma n \to \pi^- p$

$d\sigma/d\Omega$ and Σ_γ for $\gamma p \to \pi^0 p$

preliminary

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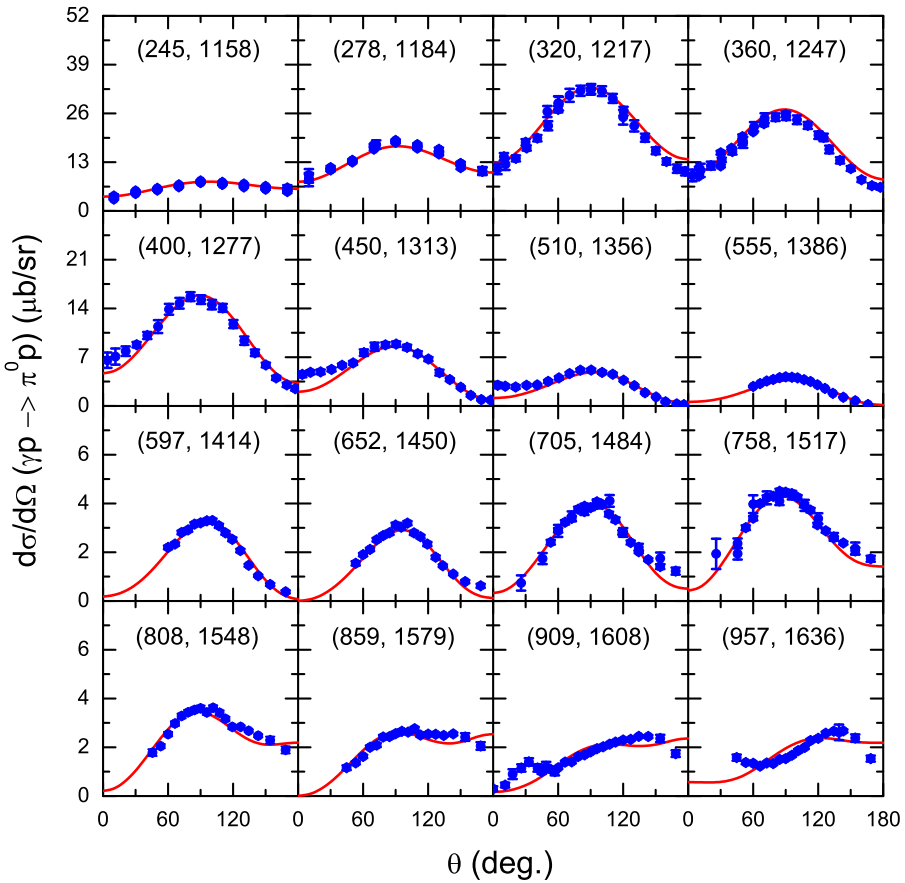
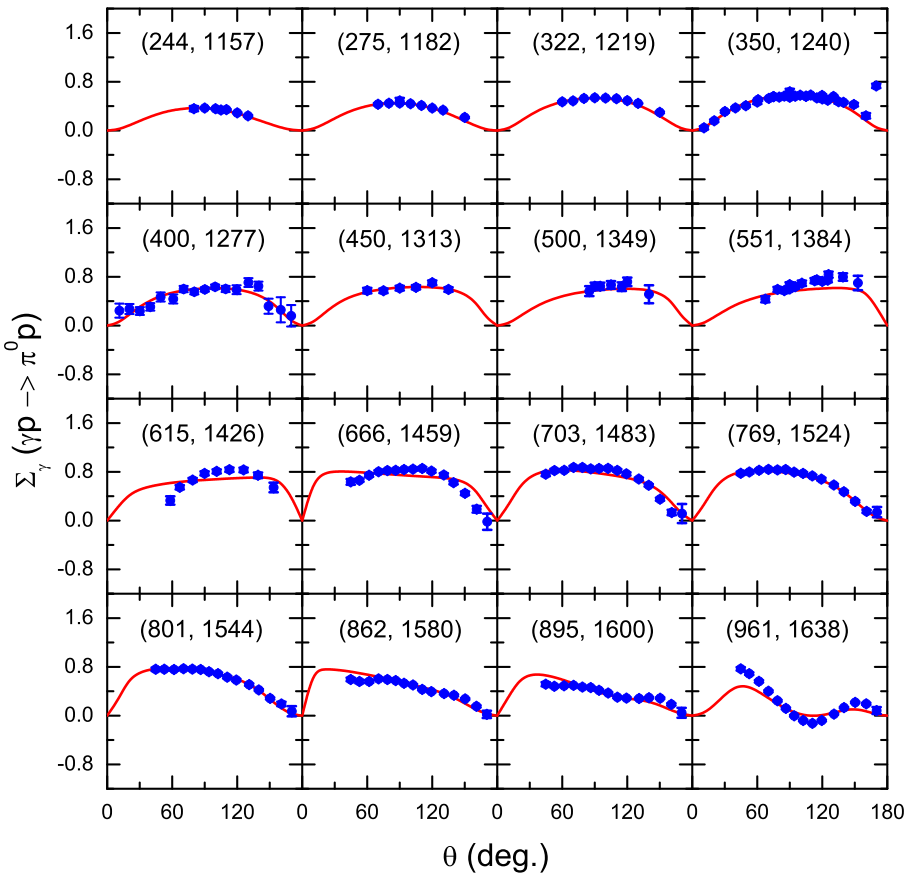


Differential cross section for $\gamma p \to \pi^0 p$



Photon spin asymmetry for $\gamma p \to \pi^0 p$

Dynamical coupled channels models in a box

Prediction & analysis of lattice data [M.D., J. Haidenbauer, A. Rusetsky, U.-G. Meißner, E. Oset, in preparation]

Discretization of momenta in the scattering equation:

$$T(q'',q') = V(q'',q') + \int_0^\infty dq\, q^2\, V(q'',q) \frac{1}{z - E_1(q) - E_2(q) + i\epsilon}\, T(q,q')$$

$$\int \frac{\vec{d}^{\,3}q}{(2\pi)^3} f(|\vec{q}\,|^2) \quad \to \quad \frac{1}{L^3} \sum_{\vec{n}_i} f(|\vec{q}_i|^2), \quad \vec{q}_i = \frac{2\pi}{L}\,\vec{n}_i, \quad \vec{n}_i \in \mathbb{Z}^3$$

$$T(q'',q') = V(q'',q') + \frac{2\pi^2}{L^3} \sum_{i=0}^{\infty} \vartheta(i)\, V(q'',q_i) \frac{1}{z - E_1(q_i) - E_2(q_i)}\, T(q_i,q'),$$

- Can be also expressed in terms of the Lüscher $\mathcal{Z}_{00}$ function up to e^{-L} relativistic corrections.
- Takes into account discretization effects of the potentials themselves.
- Twisted boundary conditions, e.g.

$$u(\mathbf{x} + L\mathbf{e}_i) = u(\mathbf{x}),\ d(\mathbf{x} + L\mathbf{e}_i) = d(\mathbf{x}),\ s(\mathbf{x} + L\mathbf{e}_i) = e^{i\theta} s(\mathbf{x}),$$

especially suited for coupled-channels problem (enables to move thresholds) [V. Bernard, M. Lage, U.-G. Meißner, A. Rusetsky, JHEP (2011)].